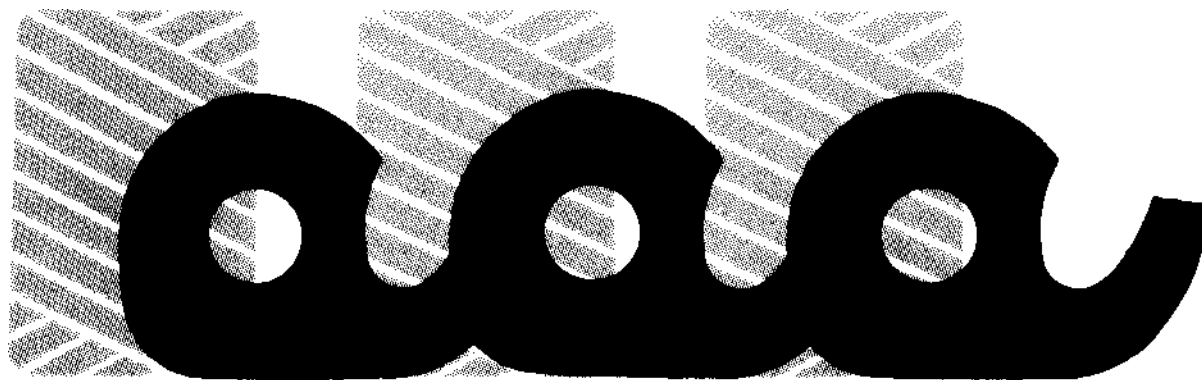




STRING & STICKY TAPE EXPERIMENTS

By R.D. Edge

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American Association of Physics Teachers**

		5=Most fundamental *=Interesting	EASY QUALITATIVE	DIFFICULT QUALITATIVE	EASY QUANTITATIVE	DIFFICULT QUANTITATIVE	Elementary School	High School	University
10.	Electrostatics								
10.01	The Electroscope	*	5			5			HU
10.02	The Versorium		4						EH
10.03	The Electrophorus		4						HU
10.04	The Faraday Ice Pail Experiment					4			HU
10.05	Capacitance					4			
11.	Magnetism								
11.01	Why is the North Pole South?		4						
11.02	Making a Magnet, and Using it as a Compass	*	4						
11.03	Lines of Force Around a Magnet			4					
11.04	Tangent Magnetometer, and Magnetic Strength of a Bar Magnet						5		HU
11.05	The Force Law Near a Bar Magnet						5		HU
11.06	Strength of Bar Magnets		2						
11.07	The Concentration of Field in a Magnet		3						
11.08	The Dip Circle						3		
11.09	What is Magnetic?	*	2						E
11.10	Penetration of Magnetism		3						EH
11.11	The Mysterious Magnet	*	5						HU

		5=Most fundamental *=Interesting		EASY QUALITATIVE		DIFFICULT QUALITATIVE		QUANTITATIVE		DIFFICULT QUANTITATIVE		
12.	Current Electricity											
12.01	The Magnetic Field Around a Coil of Wire											
12.02	The Tangent Galvanometer	*										HU
12.03	Electrical Resistance, (series and parallel)					4						HU
12.04	Magnetic Induction											U
12.05	Mutual Inductance											U
12.06	Electrolysis					3						
12.07	Forces between Parallel Conductors				4							
12.08	The Current Balance										4	HU
12.09	A Current Balance Using a Magnet									4		HU
12.10	A Simple Method of Measuring Alternating Current, and the Transformer										4	HU
12.11	The Simplest Electric Motor	*			5					2		
13.	Psychophysics											
13.01	Benham's Top	*			2							
13.02	Physical Perception - Optical Illusions	*			2							
13.03	Moiré Patterns	*			2							
13.04	Persistence of Vision	*				2						
13.05	The Ames Window	*			2							
14.	Physics Games											
14.01	Pirates' Treasure Game (Vectors)	*			4							EH
14.02	The Three-meter Dash (Kinematics)				4							
14.03	The Knee-bend Game (Energy and Power)	*			5							

		5=Most fundamental *=Interesting		EASY		DIFFICULT		QUANTITATIVE		DIFFICULT	
				QUALITATIVE						Elementary High School University	
14.04	The Wave Game	*	5								
14.05	The Kinetic Theory Game		4								
14.06	The Nuclear Reaction Game		3								
14.07	The Collision Game		4							EH	
14.08	The Electron-in-a-wire Game		3							EH	
14.09	The Close Packing Game		4							EH	
14.10	Lifting the Body - Force		4								
14.11	Reaction Kinematics	**				5				HU	
14.12	Statics-People Pyramids					4				EH	
14.13	Stand Up-Vector Forces		4							EH	
14.14	Hunker Hawser		4							E	
14.15	Conclusion-Introducing Physics to the Youngest Students									E	
15.	Miscellaneous										
15.01	Reaction - The Rocket and Balloon					2					
15.02	The Platonic Solids	*	2								
15.03	A String and Sticky Tape Digital Computer					4				HU	
15.04	Four Dimensional Tic-Tac-Toe					4				HU	
15.05	Simple Vacuum Experiments		4								

Experiment 1.01 Measurement and Error

We will start our experiments by measuring accuracy. Lord Kelvin said:

"I often say that when you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot express it in numbers your knowledge is of a meagre and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the stage of Science, whatever the matter may be."

Many schools avoid the analysis of errors - yet, in fact, it is often the most important part of the experiment - so it is dealt with here first.

Materials: paper or styrofoam cup, ruler.

Procedure: Physics is concerned with measurement. The most fundamental quantities we measure are mass, length and time. The most important feature of such measurements is their accuracy. No measurement can be perfectly accurate. For example, we can measure the speed of light to better than one part in 1,000,000*, but the age of the Universe (20×10^9 years) we only know to about a factor of two. Lay a ruler along the line below. Do not put zero against one end of the line, but put the ruler down along the line at random, paying no regard to where the ends are. (If you have no ruler, cut out the scale on page 4)

* $2.997925 \pm .000003 \times 10^8$ m/sec

Read each end of the ruler to .01 cm and put down the measurements in the table on the following page. Repeat nine times, displacing the ruler each time.

Do all the measurements agree? The difference between the separate measurements is called the error of the results. All measurements have some degree of error in them. The best value for the length is the average, which you get by adding all the results together and dividing by the number of measurements. If you add together all the differences between the average and each of the results obtained, taking all of the differences as positive, we have a measure of error. Divide this sum by the number of measurements and multiply by 1.25 (obtained from the theory of errors) to get the "standard error" for each individual measurement, which estimates how close an individual measurement will come to the mean. There is an even chance a single measurement will lie closer to the mean, or farther away from the mean, than .67 of the standard error. A class obtained an average of 10.23 cm with a standard error of .03 cm for a line similar to that above. Is your standard error close to this?

A typical example of the standard error for five measurements is shown overpage; the experimenter must have had good eyesight!

Left end cm	Right end cm	Differences	Difference From Average	Squares of Difference
2.00	12.25	10.25	0	0
2.69	12.95	10.26	.01	.0001
3.32	12.58	10.26	.01	.0001
4.34	14.58	10.24	.01	.0001
4.25	14.48	10.23	.02	.0006

Sum = 51.24

Sum of Differences = .05

Sum of Squares = .0007

Average = 10.25

Average = 0.00014

Standard error = 1.25 x .05/5 = .012.

Square Root = Standard Error = .012

DO NOT USE THIS COLUMN UNLESS CALCULATING ROOT MEAN SQUARE

	Left End cm	Right End cm	Right End -Left End	Difference from average	Squares of Differences
1					
2					
3					
4					
5					
6					
7					
8					
9					
	Sum =			Sum of Differences	Sum of Squares
	Divide sum by 9 = Average =				

Divide sum of differences by 9 and multiply by 1.25 (1.25/9 = .139) to get the Standard Error =

Divide by 9 = Mean Square

Take Square Root = Standard Error

The mean itself is not the true value, and if we divide the standard error or standard deviation as it is sometimes called, by the square root of the number of measurements, we obtain the standard deviation of the mean, which is what you see written after physical results behind the $\pm$ sign e.g.

$$5.4 \text{ inches} \pm .1 \text{ inch}$$

which means, a length is 5.4 inches, accurate to .1 inch - it could be as much as 5.5 inches or as small as 5.3 inches, but 5.4 inches is the best value. Divide your standard error by 3. You will now have the standard deviation of the mean since you obtained 9 results. Look at the picture of a meter dial (p.4). What does the pointer read? After guessing, use a ruler to check up - and show you are probably wrong. Optical illusions frequently mislead. Accurate measurement is the only way to go, and tells you how accurate any result is.

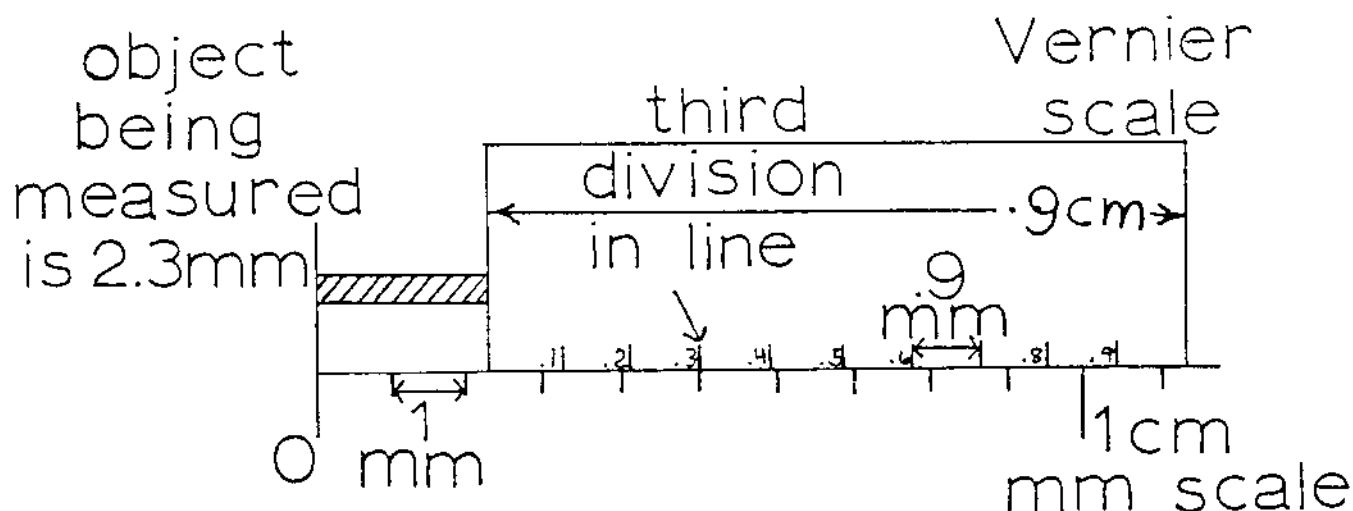
Random errors such as those above occur through the statistical fluctuations of measurements, as with throwing dice (unloaded). Systematic errors are predominantly of one sign, e.g. drift of the zero of a meter--and they affect each observation equally--as with loaded dice. If they can be determined, they can be eliminated.

The result of an observation should never be expressed to more figures than the error would suggest. For example, whereas 5.4 ± 0.1 inch is meaningful, 5.4278 ± 0.1 inch is not. The number of digits, two in the example given, is called the number of significant figures.

The conventional way of finding the standard error (or standard deviation s.d.) is to take the root mean square deviation. This is more tedious, but gives a better estimate, and should be done if you use a calculator that takes square roots. Simply square each number individually in the last column (the difference from average), add them together and divide by the number of measurements to get the mean square deviation. Take the square root of this to give the standard error for a single measurement, which we found before. The values found using the two methods will differ, but not by much.

The Vernier Scale

It is difficult to estimate a fraction of a millimeter on a ruler by eye. In order to facilitate this a Vernier scale is used. Such a scale, enlarged, is shown below.



You will see, that if the lower scale is in millimeters, the upper scale has ten divisions to nine millimeters -- i.e. each division is $9/10\text{mm}$. This means that, on moving from one division to the next on the upper scale we move along by one millimeter, but drop back also by $1/10\text{mm}$. So, if the zeroth upper division is $.3\text{mm}$ from the nearest division of the bottom scale, the first division will be $.2$, the second $.1$ and the third division will be exactly over the lower division--so the upper and lower scales are aligned. It is easy to see this, and hence to state that the length shown is 2.3mm . Now, cut out the scale below, and use it to measure the length of the line given above, by laying the zero of the millimeter scale at the zero of the line, and sliding the cut out vernier along, as shown in the picture below.



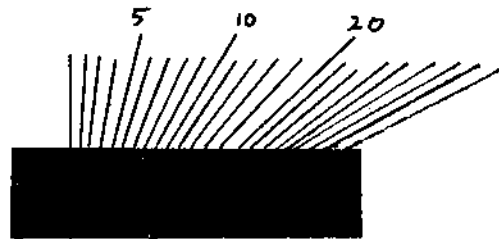
Vernier scale to be cut out



This line is 125.4 mm long.

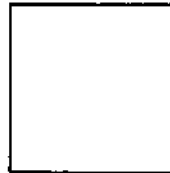
Remove the scale completely from the line, then reset it, and repeat the measurements five times. Do your measurements agree? Are they better than if you try to measure the length of the line using a millimeter ruler, estimating fractions of a millimeter, as you did previously?

Still greater accuracy can be obtained by using the same vernier to read both ends of the line, since there is an "end correction" necessary because there is an error which arises by measuring one end of the line in one way, and the other the other. If we lay the ruler along the line, slide the vernier to one end and read it, then to the other and read that, both ends are read in the same way, and the end effect cancels out. For example, if both ends were read $.1\text{mm}$ too large, the difference would still be the same in spite of the incorrect absolute value.



AREA

Length is a fundamental dimension. We can derive areas from length, for example

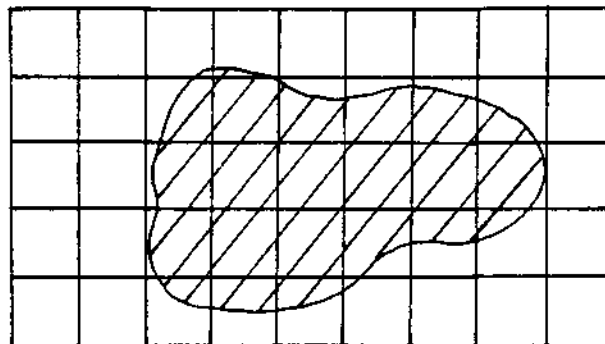


← this is one square inch, since each side is one inch.

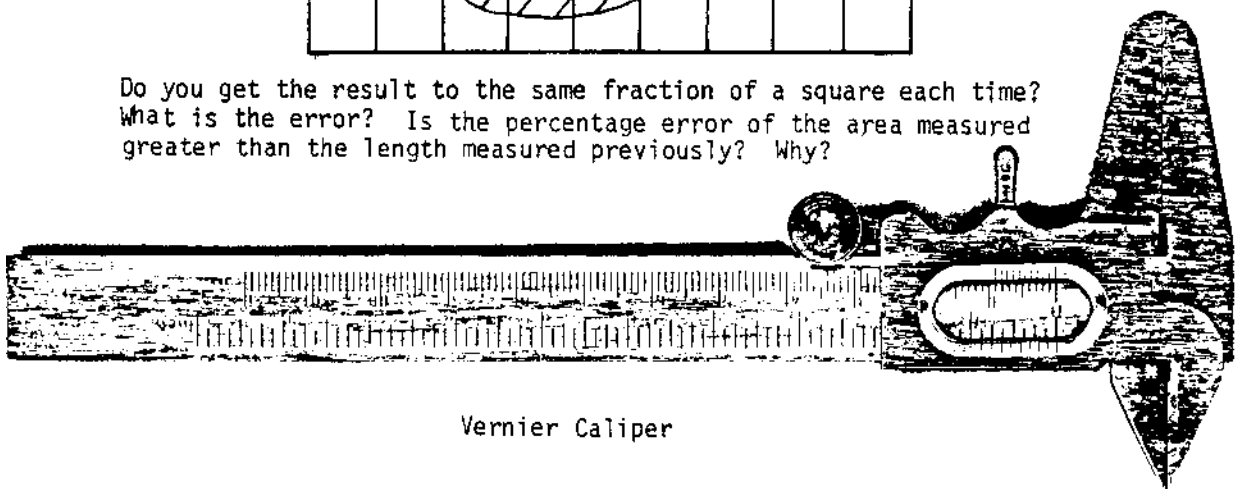


← and this is 12 sq. cm. (count them)

Add up the squares to find out how many square centimeters the cross-hatched area below is. Measure it to a fraction of a square and put down the results. Repeat twice.

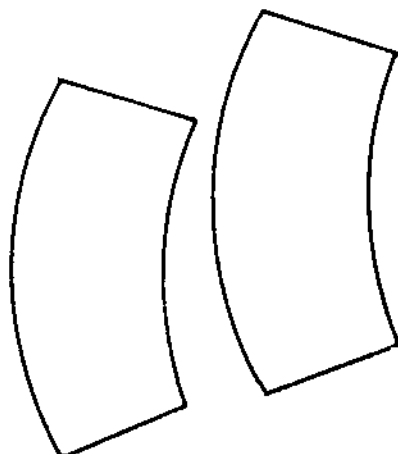


Do you get the result to the same fraction of a square each time? What is the error? Is the percentage error of the area measured greater than the length measured previously? Why?



Vernier Caliper

Although it is measured in square centimeters, an area can be any shape. Are the two areas below the same?



They are, although an optical illusion makes one seem larger -- only measurement can tell.

We see we can add squares to get area, but looking back, you will see that the area 6 cm by 2 cm gives 12 square cm. Area is length times length -- so a square 5 cm by 5 cm totals 25 cm².

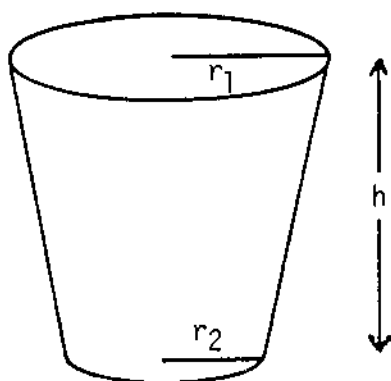
VOLUME

Now consider volume. On page 9 is a drawing with instructions how to fold it into a cube. If you look, each side is 5 cm by 5 cm, so the number of squares on each side is

$$5 \times 5 = 25$$

So each layer of the cube has 25 small 1 cm cubes in it, of volume 1cc and since there are 5 layers, this amounts to 125 cubic centimeters. So the volume is $5 \times 5 \times 5 = 125$ -- three dimensions of length multiplied together. But again, the volume does not have to be a cube or rectangle. Fill the little cube with water and empty it into the Dixie cup. Has the volume changed when the water pours into the Dixie cup? Make a mark on the side of the cup to which the water fills it, and repeat with another cubeful. This will be $125 + 125 = 250$ cc and you have a 250 cc graduated vessel. Since 1 cc of water weighs 1 gm, your cup will have a mass very nearly 250 gms, and you can use it as a calibrated mass as well as volume. You could weigh your marbles against this to find their mass. The whole point is, using just the sheet of paper, and a little water, we end up with standards of mass and volume.

The volume of a paper or styrofoam cup is $\frac{h}{3} (r_1^2 + r_1 r_2 + r_2^2)$ where r_1 and r_2 are the inside radii of top and bottom and h is the length. Most styrofoam cups have a volume of about 200cc when filled level with the brim. Some cups are labeled on the bottom 6 oz. or 180cc, and hold this much when filled just below the brim.



A sheet of paper $8\frac{1}{2}$ " square makes a box which holds 180 cc. Fill and empty the box rapidly - unless you use wax paper, the water quickly makes it soggy.

Masses - The following table was obtained by the Bureau of the Mint. Note before 1964, when clad coins were introduced, the mass of a half dollar was twice that of a quarter, which was 2.5 times that of a dime, whose mass was half that of a nickel. A dollar bill has a mass close to 1 gm.

Penny	before 1942	3.110 g
	1943	2.700 g
	1944-1982	3.110 g
	1982	2.500 g
Nickel		5.000 g
Dime	before 1964	2.500 g
	1965-date	2.268 g
Quarter	before 1964	6.250 g
	1965-date	5.670 g
Half-dollar	Except the silver-clad Washington Bicentennial	5.750 g
	before 1964	12.500 g
	1965-1970	11.500 g
	1971-1974	11.340 g
	1977-date	11.340 g
	Silver-clad Kennedy Bicentennial	11.500 g
	Cupruo-nickel-clad Kennedy Bicentennial	11.340 g

The marbles used in our experiments vary in mass from 4.8 to 5.8 gm. Marbles taken from one packet rarely vary by more than 10% in mass. An average of 5.4 gm is probably a good choice.

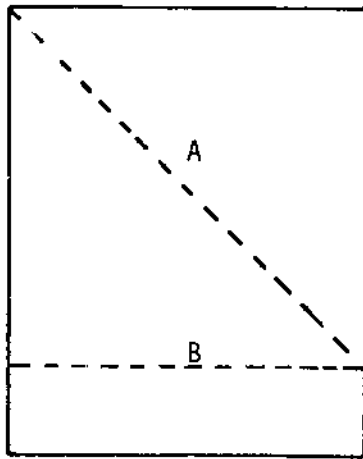
Now that we know how to measure length and mass, how do we measure time? We can do so in terms of length through some standard derived unit such as velocity ($\frac{\text{length}}{\text{time}}$) or acceleration ($\frac{\text{length}}{\text{time}^2}$).

TIME

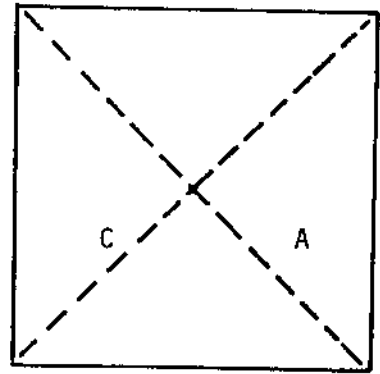
Light has a constant velocity in all systems, so it provides the best standard -- but it is inconveniently fast -- the time light takes to travel a foot is 10^{-9} seconds, a nanosecond, so we could call a foot a light - nanosecond. The most convenient standard is the earth's gravitational field, which provides an acceleration, g , of 981 cm/sec^2 . A simple pendulum has a period (the time for the pendulum to swing to and fro) of T given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where L is the length of the pendulum. So a pendulum 99.4 cm long takes one second for half a complete oscillation, i.e. from one side to the other. One's ear is very sensitive to frequency - so using a rhythmic song ("onward Christian Soldiers" has been used by photographers who must time in the dark) one can time quite accurately.

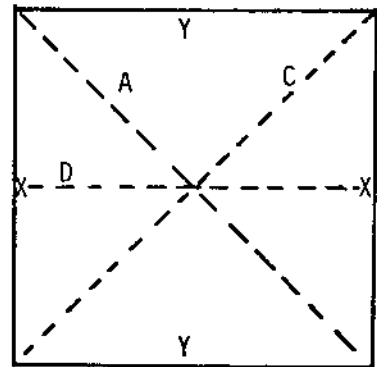


1. Fold along line A. Cut along line B.



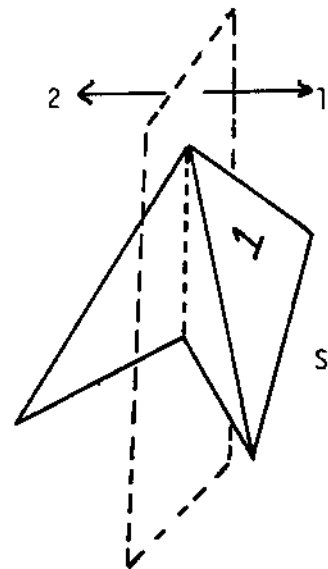
2. Open up paper and fold along line C.

3. Open up paper, turn it over and fold along line D. Then unfold the paper. Now turn the paper over again. Bring the two X marks together and then bring the two Y marks together. Your paper should now look like the two diagrams below.



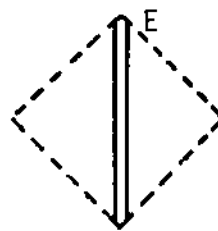
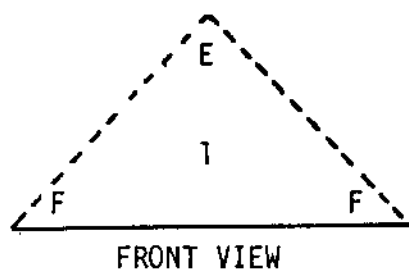
FRONT VIEW

4. You now have two symmetrical sides, sides 1 and 2, as shown in the side view diagram. THE NEXT PAGE OF INSTRUCTIONS PERTAIN TO SIDE 1. AFTER COMPLETING EACH INSTRUCTION TURN THE PAPER OVER AND DUPLICATE THE INSTRUCTION FOR SIDE 2.

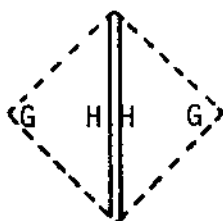


SIDE VIEW

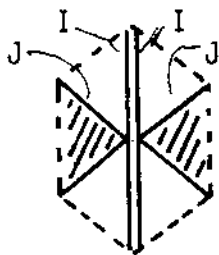
REMEMBER TO DUPLICATE THE INSTRUCTIONS FOR SIDE 2.



5. Fold points F to point E.....Your paper should look like this.



6. Fold points G to H.....Your paper should look like this.



7. fold leafs marked I into slots marked J. Your paper should look like diagram 8.

8. Blow into K to make a cube.

EXPERIMENT 1.02. Probability and the normal curve of error. It is important to know not merely the average error, but also the distribution.
Materials: Eight coins

Procedure: Unless you have a double headed penny, each coin has two different sides. If you toss one coin, what is the chance, or probability it will come down heads?

Now toss two coins together. Is there the same chance they will come down both heads, both tails, or one head and one tail?

Both heads: the chance of one head is $1/2$, so the chance of two is $1/2 \times 1/2 = 1/4$. HH - one chance in four or a probability of $1/4$.

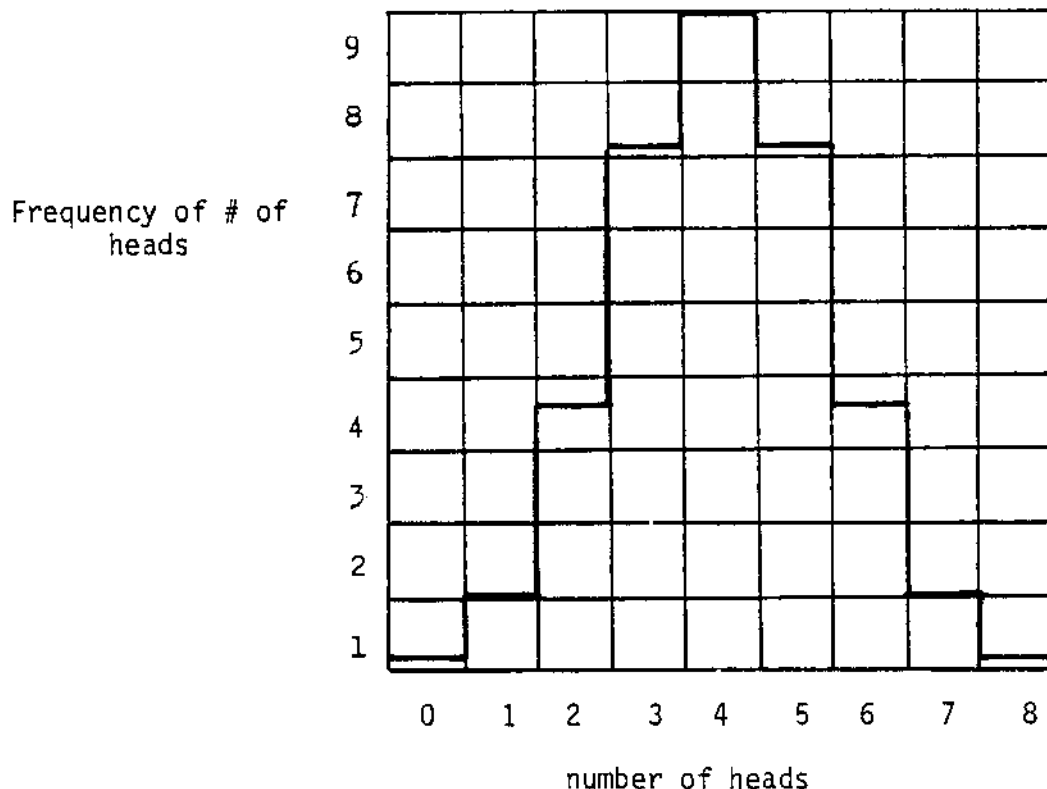
Head and tail - not $1/4$, but $1/2$, since HT is the same as TH, i.e. the order doesn't matter.

Both tails ($1/4$)

In practice, you rarely get exactly one quarter of the throws to be both heads, but the more tosses you make the closer it gets to the expected value.

Now, toss eight coins together. Count the number of heads and put an X in the corresponding column of the table below. Repeat this a large number of times, making an X in the next square above if there is already an X in the column, thus

X
X



More advanced students note that the binomial expansion, whose terms give the number of heads and tails for the eighth row, takes the form

$$(1+1)^8 = 1 + 8 + \frac{8 \times 7}{1 \times 2} \dots$$

Generally

$$(1 + 1)^N = 1 + N + \frac{N(N-1)}{1 \times 2} + \frac{N(N-1)(N-2)}{1 \times 2 \times 3}$$

where there are N rows (or N coins to be tossed).

The sum of the number of trials is $2^8 = 256$ for the eighth row (in general 2^N). If we let N become very large, the distribution takes the form that the number of times we get $\frac{N}{2} - n$ heads and $\frac{N}{2} + n$ tails is given by

$$S = 2^N \frac{\sqrt{2}}{\pi N} e^{-\frac{2n^2}{N}}$$

where the standard deviation $\sigma^2 = \sum \frac{sn^2}{N} = N$.

This approximation is good for large N, but very poor for small N. For example, in the case where $N = 8$, it gives 13, 20, 27, 36, 34, 27, 20, 13 which does not agree with the previous results, however as N becomes greater than 100, the distribution sharpens very rapidly, and the approximation becomes very much better for very large N.

A distribution $\frac{1}{\sigma} \frac{\sqrt{2}}{\pi} e^{-\frac{2x^2}{\sigma^2}} \Delta X$ where $\sigma^2 = \sum \frac{(X-\bar{X})^2}{N}$ (not necessarily N)

is called a Gaussian distribution, or "normal curve of error", and represents the probability an event is between X and $X + \Delta X$.

Experiment 1.03. Vectors

Materials - Pencil, ruler, scissors

Procedure - PART 1: Distance

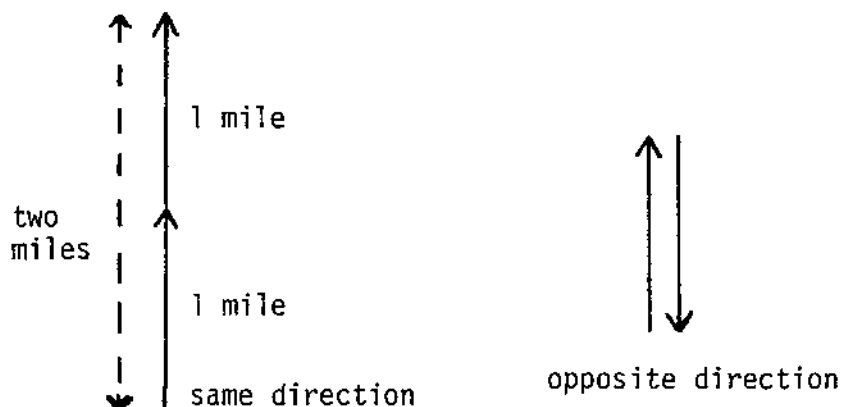
Quantities such as mass are called "scalars." They can be defined by a simple number, telling us how much we have - if we have 2 kg, we have twice as much as 1 kg. However, some quantities, such as length, require a direction as well to specify them. For example, if we say we walked a mile, we might well be asked where, or in which direction - East, West, North or South. We might have walked half a mile north, and half a mile south, and arrived back at the same spot - or walked one mile north - so clearly we must specify direction together with distance. Direction must be carefully specified too. - For example -

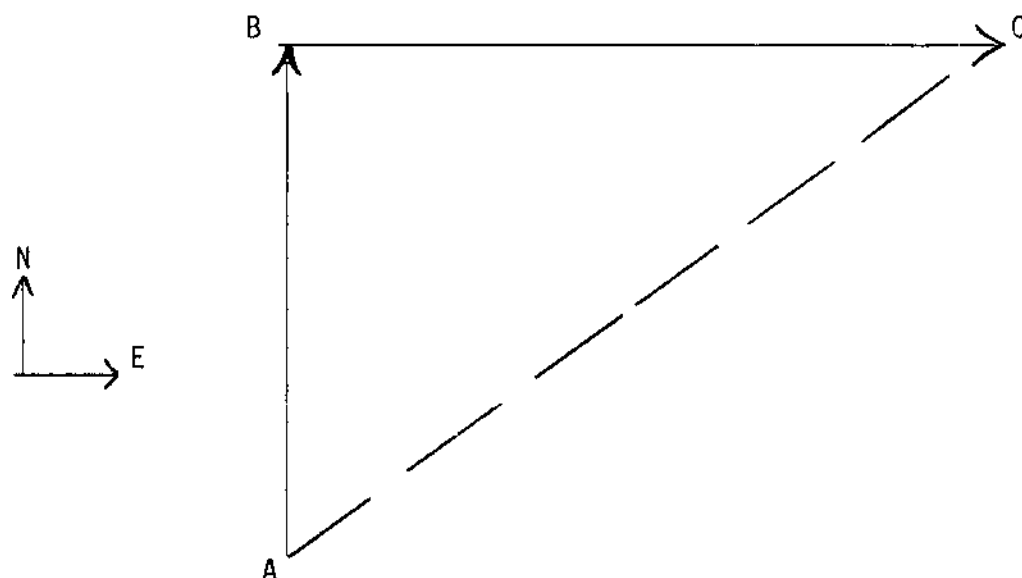
A man walked one mile south, one mile west, and one mile north, arriving back at the place he started, where he saw a bear. What color was it?

Answer - White. The only place you can walk one mile south, west and north, and arrive back where you started is the north pole, so it must be a polar bear. But note, this could only happen on a sphere, such as the earth - it would be impossible on a plane.

Now, scalars add arithmetically, but vectors do not - if we walk one mile, and then another mile, we may be two miles from where we started, walking in the same direction, or back at the start, if in opposite directions.

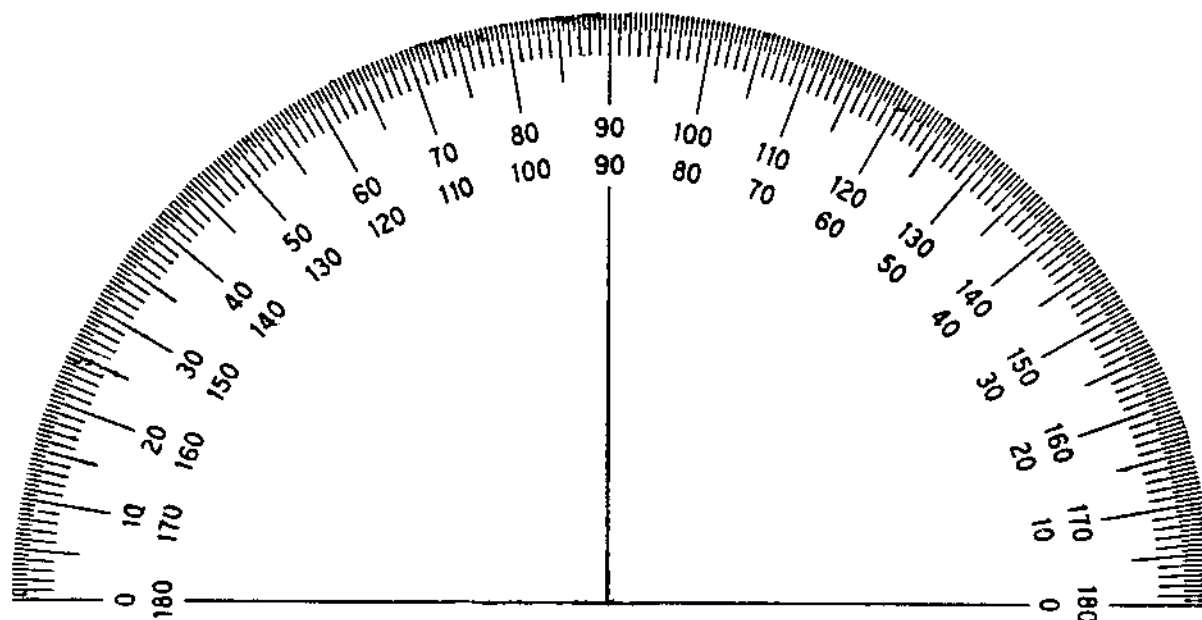
The easiest way to talk about vectors is to draw a diagram. For example, if we let one inch equal one mile, our two walks might look as follows



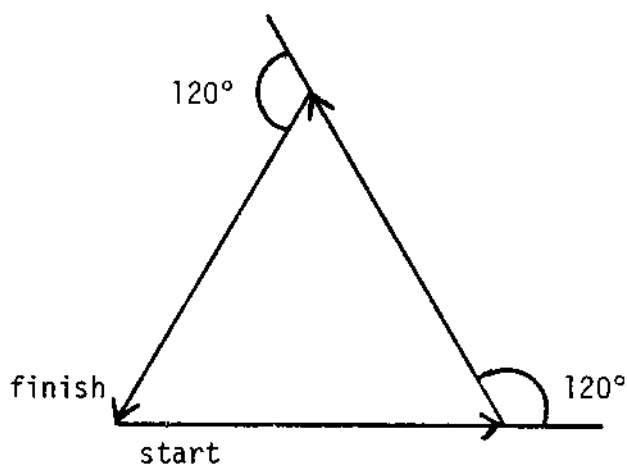


Now, suppose you walk three miles north, and four miles east. We measure three inches north AB on the paper, as shown, and four inches east BC. Then the distance from where you started is AC. Measure it. How far are you from where you started? You are five miles, which is not the arithmetic sum of three plus four. Now try this one out yourself. You walk two miles north and two miles west. Draw it below, but use a scale where 1 cm represents 1 mile. How far are you from the place where you started? The answer should be about 2.8 cm.

Unless we are walking N, S, E or W, it is generally easier to talk about direction in degrees. Cut out the protractor printed below, which is graduated in degrees.



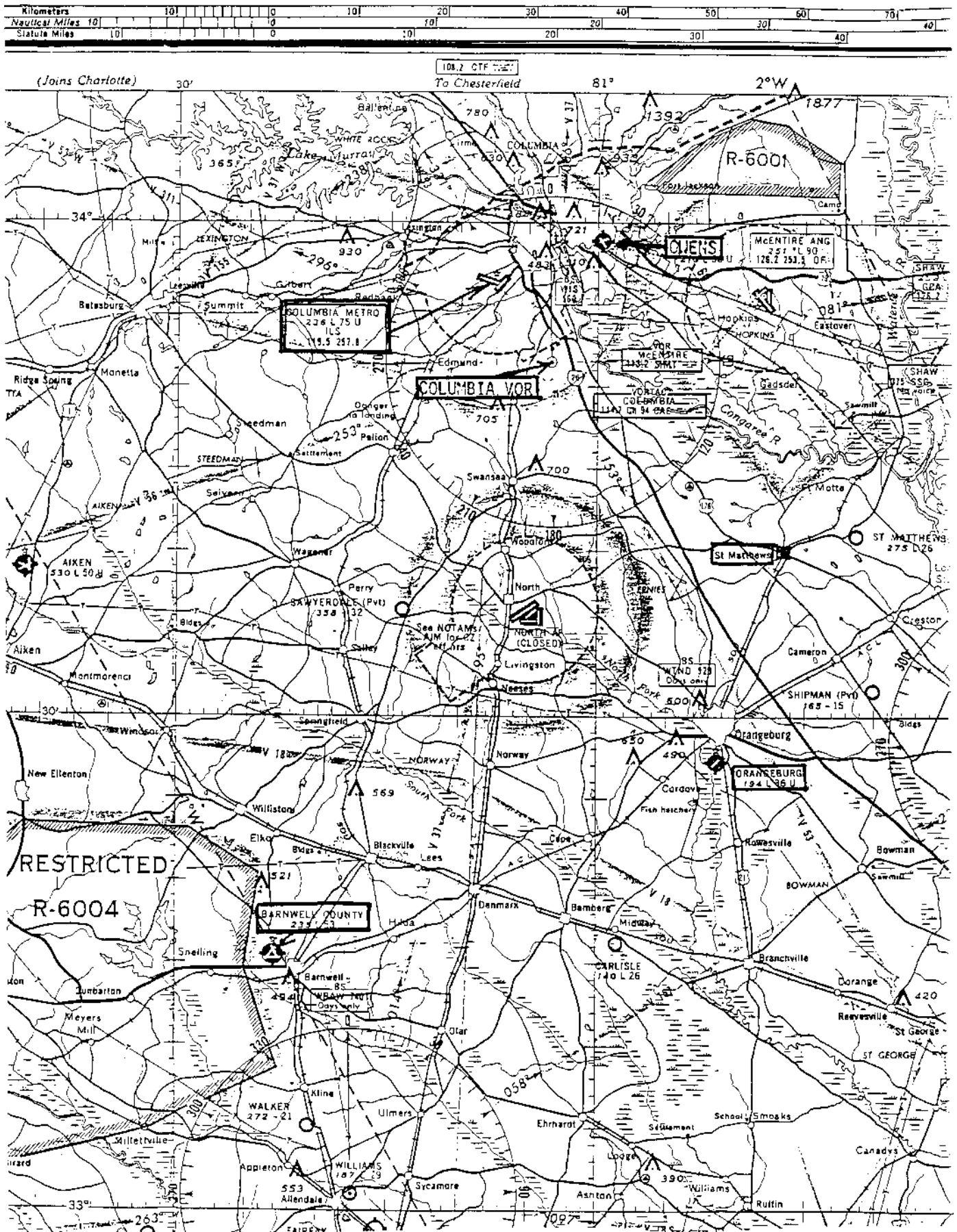
Now, suppose we walk one mile, turn 120° to the left, walk another mile, turn 120° left, and walk a third mile. We draw a map or scale diagram of this as shown below - and you will see you are back at the origin.



On the following page is a "sectional map" which airmen use to navigate. The scale of miles is given along the top of the chart, and may be cut off for measurement. If you start from Owens, the airport marked at the top of the map ($33^\circ 59' 81^\circ$), and fly due south 22 statute miles, then due west 40 miles, at which airport will you be?

Notice there is a circle of angles marked around the Columbia VOR (which is a radio beacon used for navigation). Suppose you fly over the VOR, head to 249° and travel 39 statute miles; which airport will you be over?

If you fly 138° from north, starting at Columbia Metropolitan Airport, and travel 39 miles, which private airport will you be over?

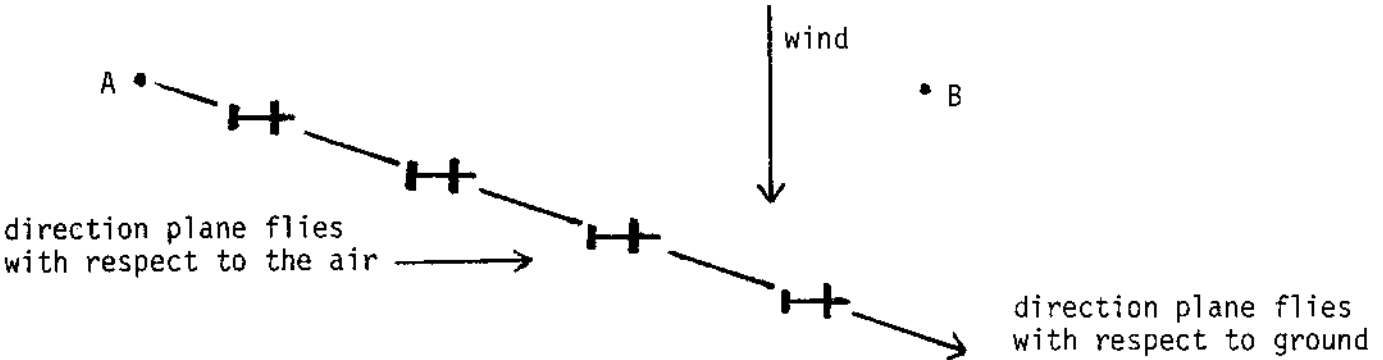


PART 2: Velocity

Not only displacements are vectors, but anything associated with distance, such as velocity, or acceleration. Since force is mass times acceleration, force is also a vector quantity. This we can see more intuitively, since the direction an object moves depends on the direction we push it--the direction the force acts.

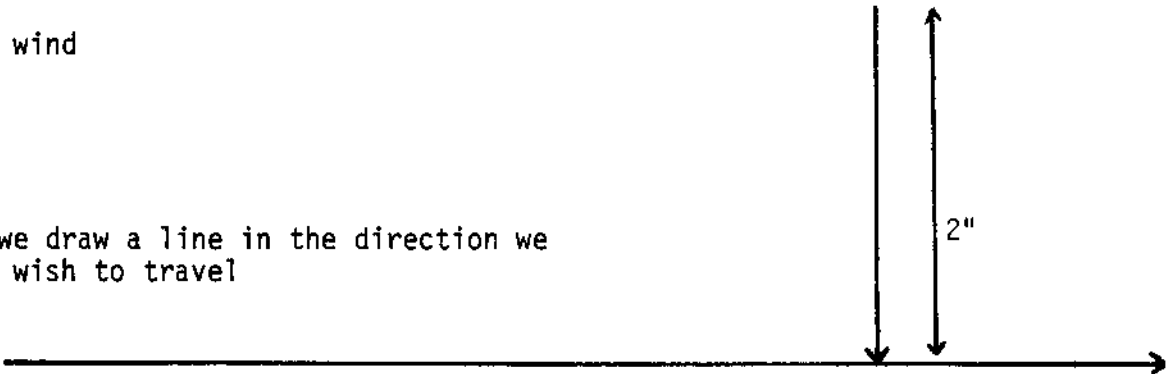
To see how velocities add, let us take the example of an airplane flying between two points A and B.

It might seem reasonable, once in the air from A, to point the airplane at B - but this might be very bad, for if there is a side wind, as shown below, a plane would not end at B, but somewhere else.

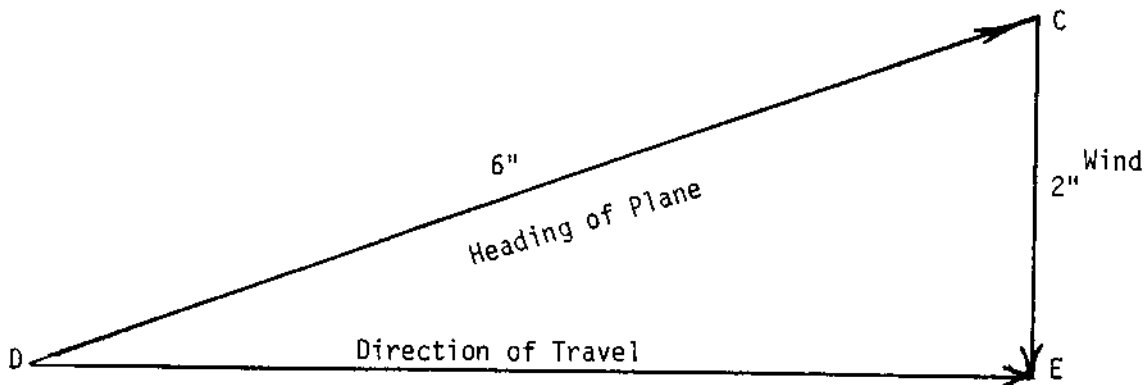


We must arrange that the velocity of the plane plus the velocity of the air takes it to B. For example, if there is a twenty mile an hour cross wind, as shown above, and the plane flies at sixty miles an hour, we make a scale drawing in which one inch represents ten miles an hour. First we draw the cross wind

Then we draw a line in the direction we wish to travel



Then, we mark off a distance from C equal to the planes velocity, that just meets the direction of travel



As with distance, the vector sum of the two velocities is the direction we wish to travel. Measure the length D E. This represents the velocity with respect to the ground and is 55 mph.

To summarize: If you know the crosswind, the direction of the plane on the ground, and its airspeed, proceed as follows:

- 1) Draw a line in the direction of ground travel.
- 2) Draw a second line starting from the first, in the direction from which the wind comes, of length representing the speed of the wind.
- 3) Starting from the far end of the wind line, draw a line of length representing the airplane speed, to meet the line of ground travel at a point p.
- 4) The distance from p to the point where the wind line starts represents the ground speed of the plane.
- 5) The direction of the line joining the ground line and the wind line represents the plane's heading in the air.

Now try this example, using 1 cm = 10 mph.

Wind from the south of 40 mph
 Plane's airspeed is 100 mph
 Direction of travel with respect to ground - east
 What is the plane's heading and ground speed?

A simple example arises if you are given the plane's direction in the air, airspeed, and the windspeed. Then you draw a line of length and direction the speed and heading of the plane, from the end of which draw a line of length and direction the windspeed, and the third side of the triangle is the ground speed and direction of the plane on the ground.

If a plane flies at sixty miles an hour into a headwind of sixty miles an hour, what is its velocity with respect to the ground? Well, it wouldn't travel very far, I can assure you.

1. You are cleared to runway 9 of Barnwell County Airport (runway heading, i.e. the direction the plane faces, is in tens of degrees from north, so runway 9 is 90^0). What is your ground speed just after takeoff (at 50 ft. altitude) if the gross weight of the aircraft is 2,000 lb. with a head wind of 15 knots (use the takeoff data to find indicated air speed, i.a.s., the speed of the plane with respect to the air).
2. Immediately after takeoff, you head for Orangeburg. What is the bearing (direction from North) of Orangeburg from Barnwell?
3. If you fly your aircraft at an altitude of 10,000 ft., what heading should you have to arrive at Orangeburg, at an air speed of 108 mph?
4. How far is Orangeburg from Barnwell?
5. If you leave Barnwell at 12:00 noon, at what time will you arrive in Orangeburg?
6. Should you refuel in Orangeburg to arrive in Columbia? Show your calculation. (The plane had 42 gal. of fuel and consumes 6 gal./hour.)

I.A.S. (Indicated Air Speed) for take off

Gross wt.	I.A.S.
1700	75 mph
2000	79
2350	84

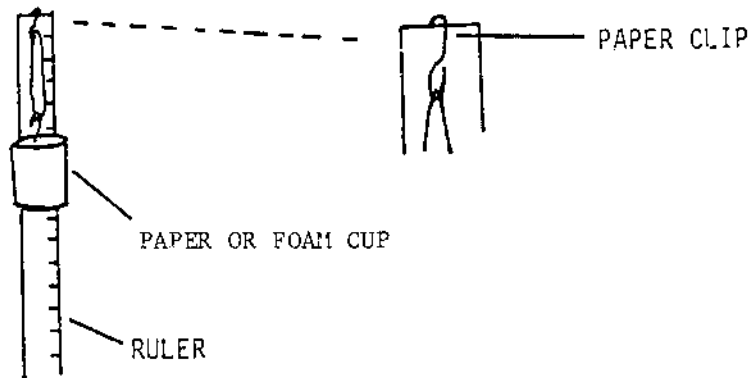
This is the air speed read in the plane before it can lift off the ground carrying the weight shown.

7. What is the bearing of St. Matthews from Orangeburg?
8. You fly the rail track to St. Matthews. At what angle to the rails should you fly, if your air speed is now 90 mph? (wind speed and direction 10 knots at 100^0)
9. You turn over St. Matthews and make for the Columbia VOR $33^0 52'$, $81^0 3'$. What is your heading, and groundspeed, if the airspeed is 90 mph?
10. Flying directly to Columbia Airport, how many minutes would it take from the VOR?
1. The ground wind at Columbia is now 10 mph from the northwest. You make a downwind approach to runway 29. What angle to the runway should you fly if your airspeed is 80 mph?
2. What is the total time for the trip, exclusive of the time on the ground?

Experiment 1.04. The Rubber Band Balance (Hooke's Law) and Stored Energy

Materials needed: ruler, several marbles, rubber band, two paper clips, cup

Procedure: Support the rubber band by a bent paper clip, as shown from the top of the ruler. Add marbles, one by one, to a cup, attached to the other end by another paper clip. Measure the length of the rubber band each time in centimeters, putting 0 cm at the top.



Put the length for each marble on the table.
An example is given below.

EXAMPLE

number of marbles	reading on ruler	difference from 0 marbles
0	15.1	0
5	15.6	0.5
10	16.4	1.1
15	18.0	1.6

[illegible]

Plot the difference (which is the extension of the rubber band) on the graph over, where a sample plot is also given. Are your results similar to the sample? Can you draw a straight line through your points? What does this show?

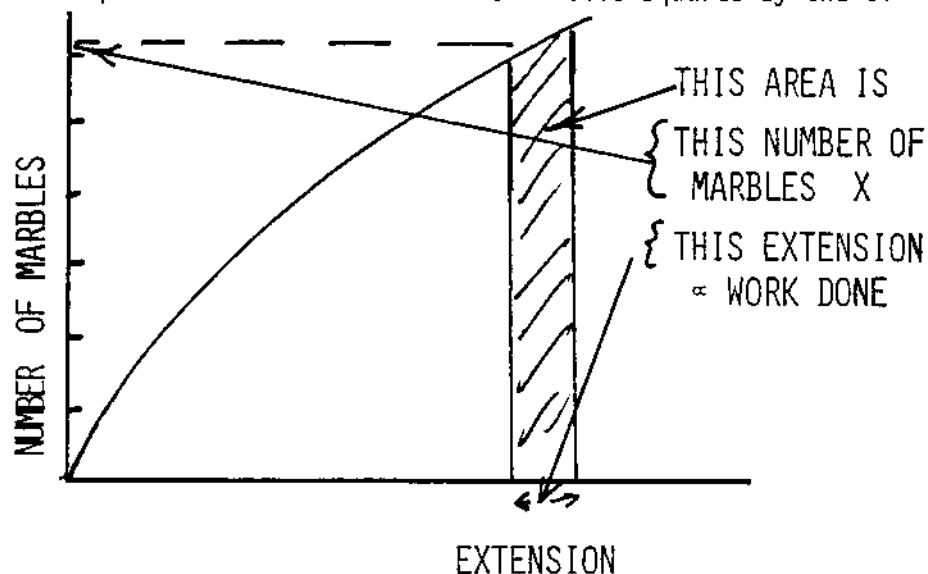
Qualitatively: The rubber band stretches uniformly with the increase in weight, which is the force of gravity on the board (mass $\times$ acceleration due to gravity). i.e. the extension is proportional to the number of marbles, and the plotted points lie on a straight line. This proportionality is known as Hooke's law. However, as the load increases, the extension becomes greater than predicted by this law, and the points diverge from a straight line. Hooke's law is no longer obeyed. Do you get the same values for the extension when you repeat the experiment? Probably not--rubber bands are not very reproducible.

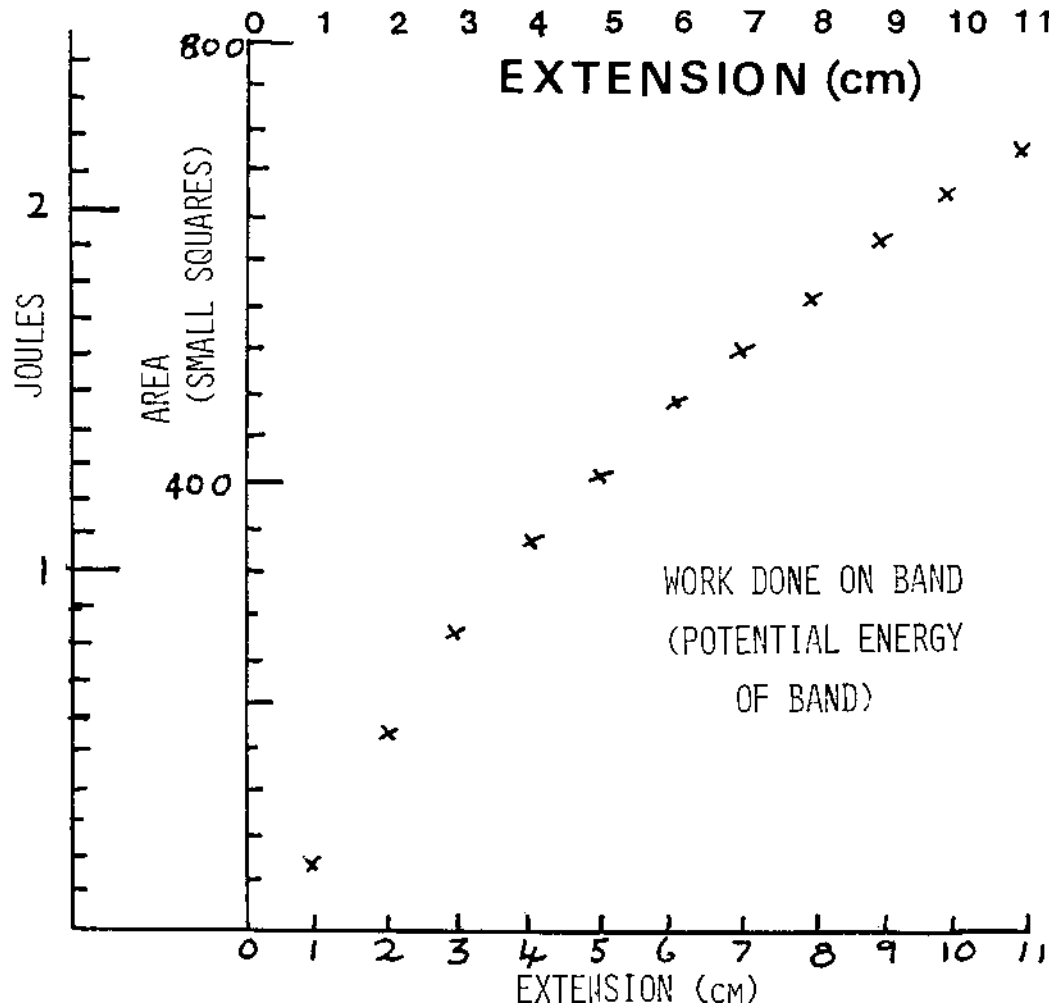
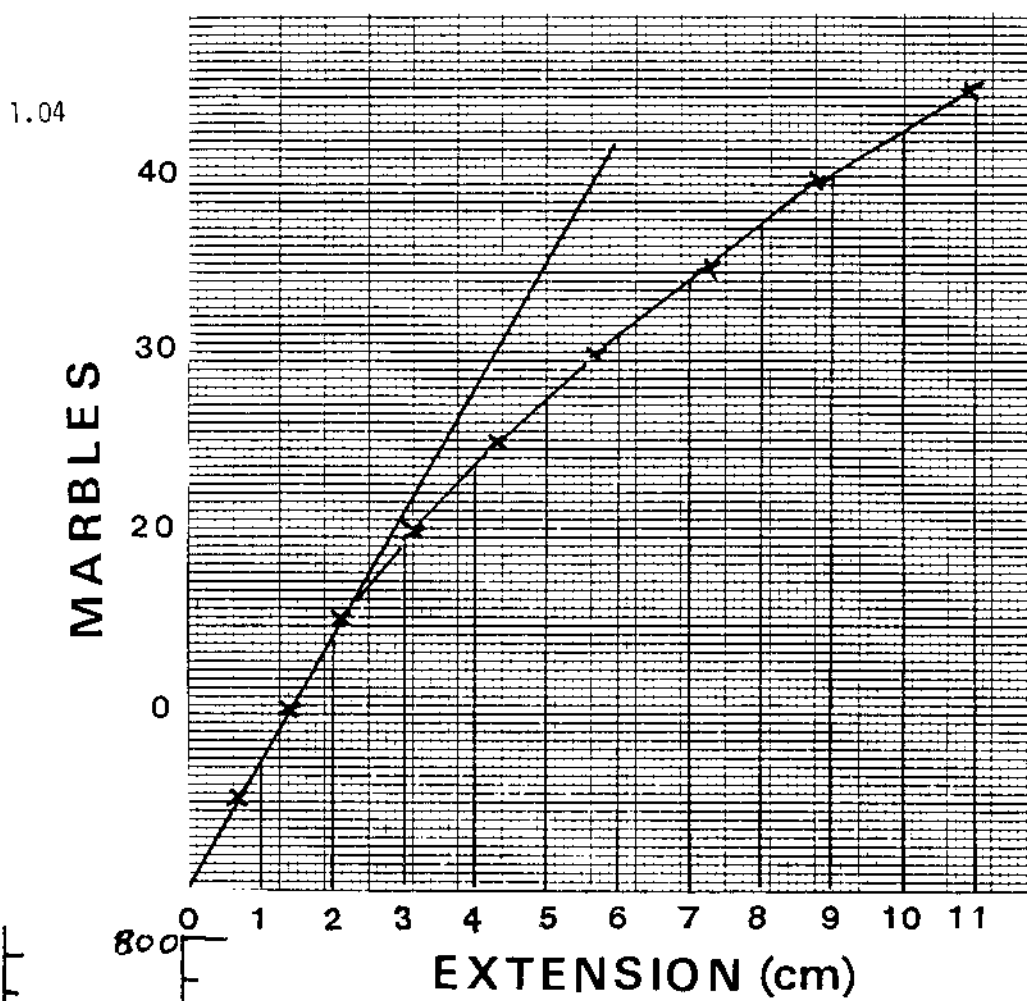
Quantitatively: We can use the rubber band to weigh objects, now we have calibrated it with a graph--see how many marbles are in the cup by reading the scale. Does this agree with counting the marbles? How accurate is your weighing machine? Do not be too surprised if it is not very accurate--rubber bands behave differently after they have been stretched. The effect that the band does not recover immediately after being stretched is called "hysteresis".

Stored Energy: The marbles do work in extending the rubber band. Work is force times distance. However, since the force is mg , and the number of marbles increases as the rubber band extends, the work done to extend the band the first centimeter is much less than that required to extend it the fifth. This work is stored in the form of potential energy of the rubber band. We would like to plot the energy stored against the extension of the rubber band. The force required to extend the rubber band a small distance is reasonably constant--the work done will be the force times this extension.

In our plot of number of marbles against the extension, the force times a small extension will be proportional to the area under the curve which lies above this extension (see figures) since this is the number of marbles times the extension.

The Number of little squares for each increase of a centimeter is plotted in figure below. Do the same for your curve. Now, since there are 4 little squares for 1 cm and 1 marble, and one marble weighs approximately 5 gm, so the work represented by these squares is $.005 \times 9.81$ Joules, and one little square represents .0123 joules. Hence, we can replace the scale of number of little squares by one of Joules, as shown.

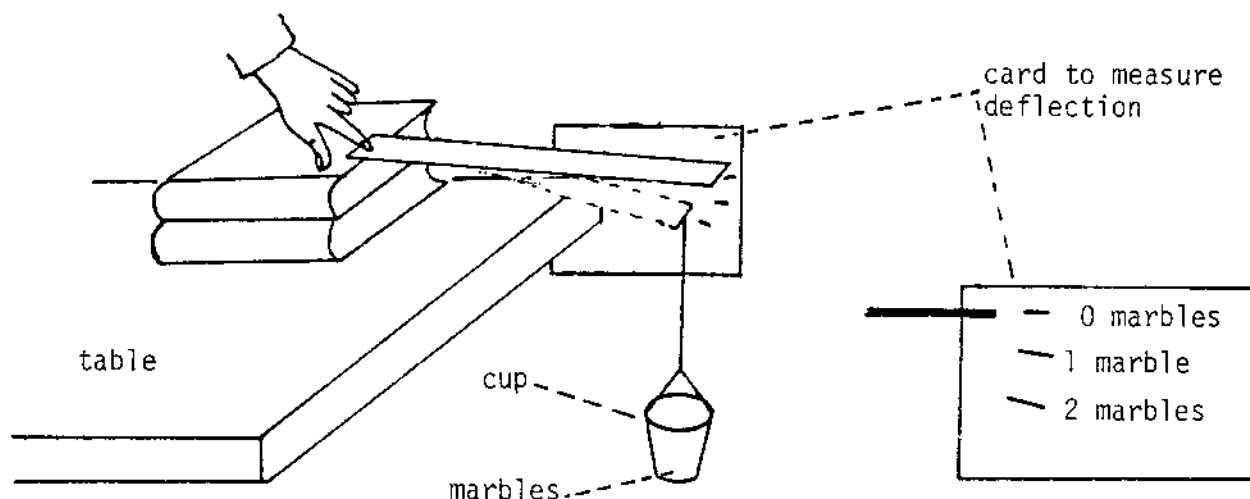




Experiment 1.05. Elastic Forces

Materials: ruler, piece of card, book support, paper cup, marbles, pencil.

Procedure: Attach a paper cup by a string to the end of the ruler. Hold down the other end of the ruler, by its last two inches, to the book support very tightly as shown.



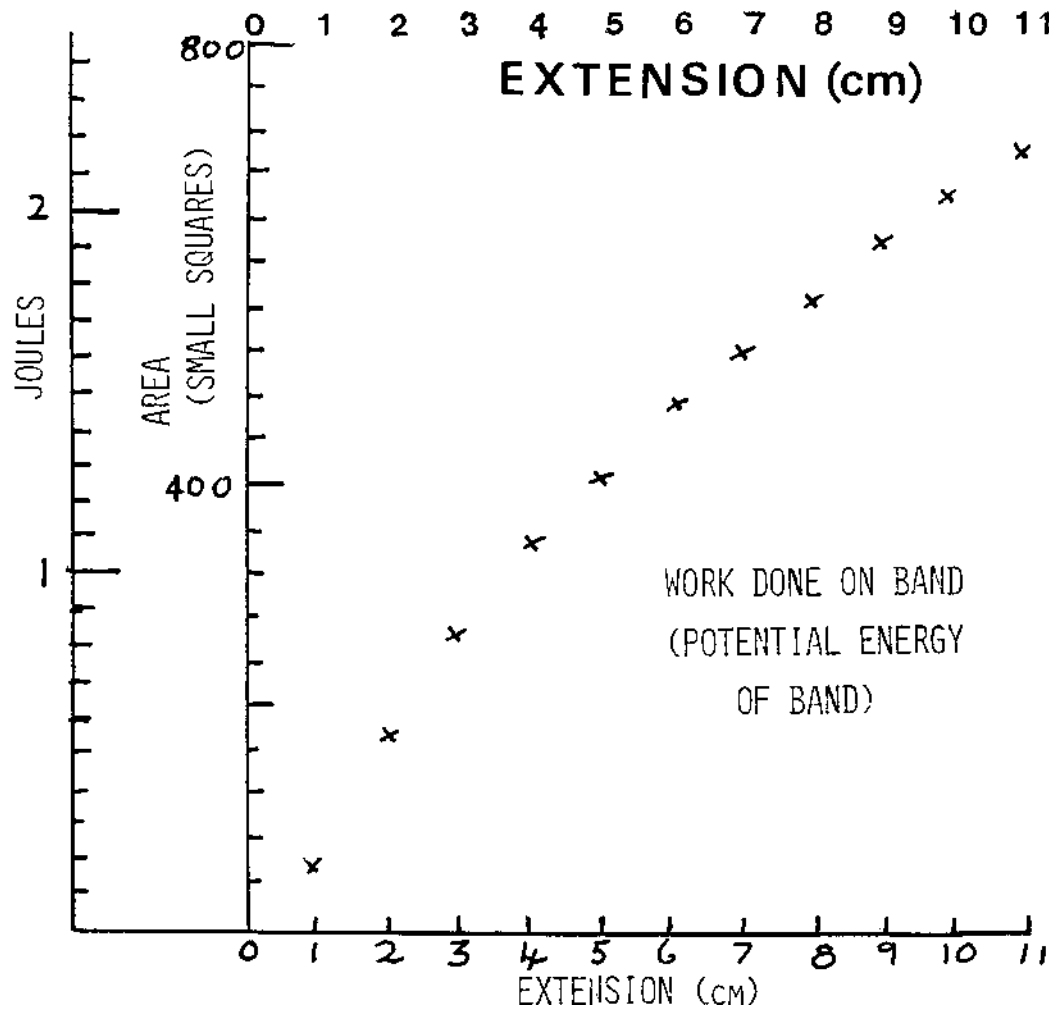
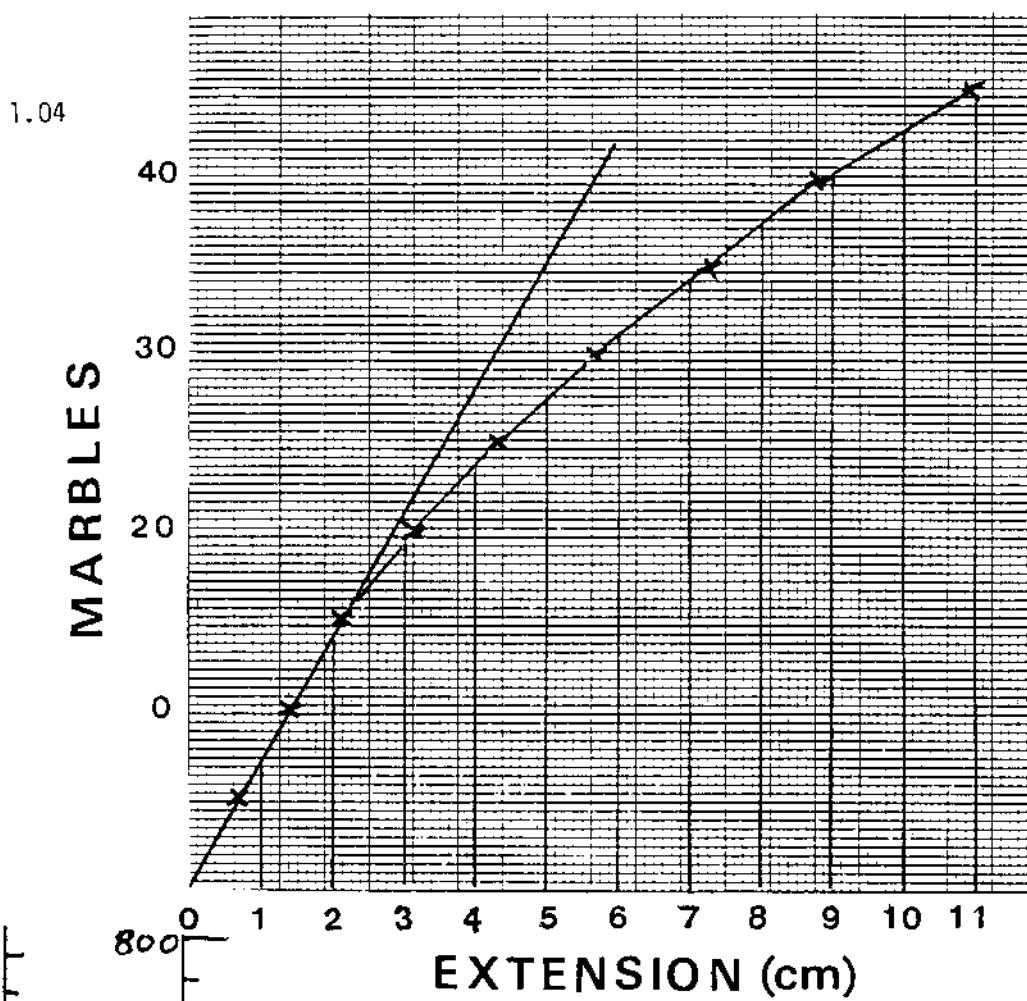
Now using a piece of card, mark on the card the deflection of the end of the ruler with different number of marbles in the cup. A second ruler can be used instead of the card, or the scale below.

Qualitative Questions: Does the ruler go back to its original position when the marbles are removed? What would happen if you kept on adding marbles? Does the deflection go up or down with the number of marbles?

Quantitative: Plot the deflection against the number of marbles. Can you draw a straight line through all the points?

Experiment 1.32 shows how gravitational mass, measured in this experiment, is related to inertial mass.

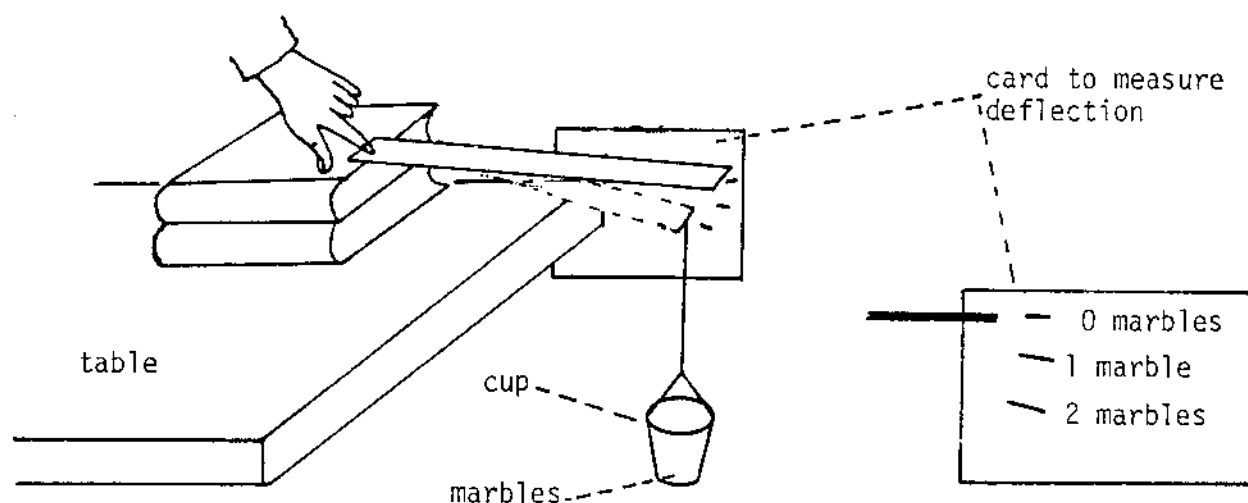




Experiment 1.05. Elastic Forces

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Procedure: Attach a paper cup by a string to the end of the ruler. Hold down the other end of the ruler, by its last two inches, to the book support very tightly as shown.



Now using a piece of card, mark on the card the deflection of the end of the ruler with different number of marbles in the cup. A second ruler can be used instead of the card, or the scale below.

Qualitative Questions: Does the ruler go back to its original position when the marbles are removed? What would happen if you kept on adding marbles? Does the deflection go up or down with the number of marbles?

Quantitative: Plot the deflection against the number of marbles. Can you draw a straight line through all the points?

Experiment 1.32 shows how gravitational mass, measured in this experiment, is related to inertial mass.



What does this show?

A force for which the deflection is proportional to the magnitude of the force is called an "elastic force". The "spring constant" is the constant of proportionality between the forces and the deflection. If we use the unit of mass as 1 marble, what is k ?

force = $k \times$ deflection

g = gravitational constant = 9.81 m/sec^2

$k = \frac{g \times \text{number of marbles} \times \text{mass of marbles}}{\text{deflection}}$

k generally turns out to be about 39 N/m .

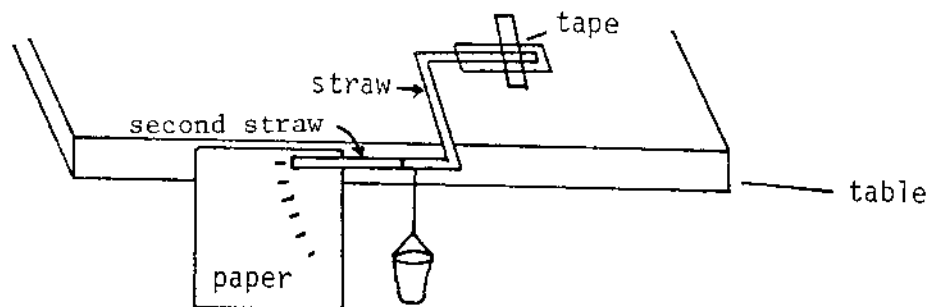
You can go on to Experiment 1.38 for another method of finding k .

Number of Marbles	Deflection
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	
21	
22	
23	
24	

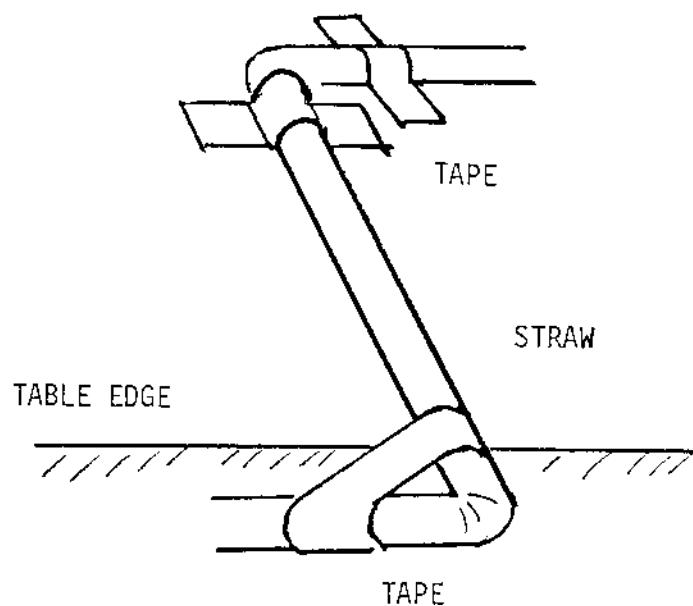
Experiment 1.06 Torsion Balance

Materials needed: Two drinking straws, marbles, dixie cups, string, paper clip.

Procedure: - Bend about two inches of a drinking straw through a right angle, and tape it so that about three inches overhang the table, as shown.



Bend at right angles, so the end lies along the edge of the table, cut the end of the second straw, and slip it in the first. Tape a piece of paper against the table edge. Hang the cup from the end of the straw as shown. Add marbles, one by one, and mark the paper where the end of the straw lies. You can now tell how many marbles are in the bucket by reading the scale you have made.

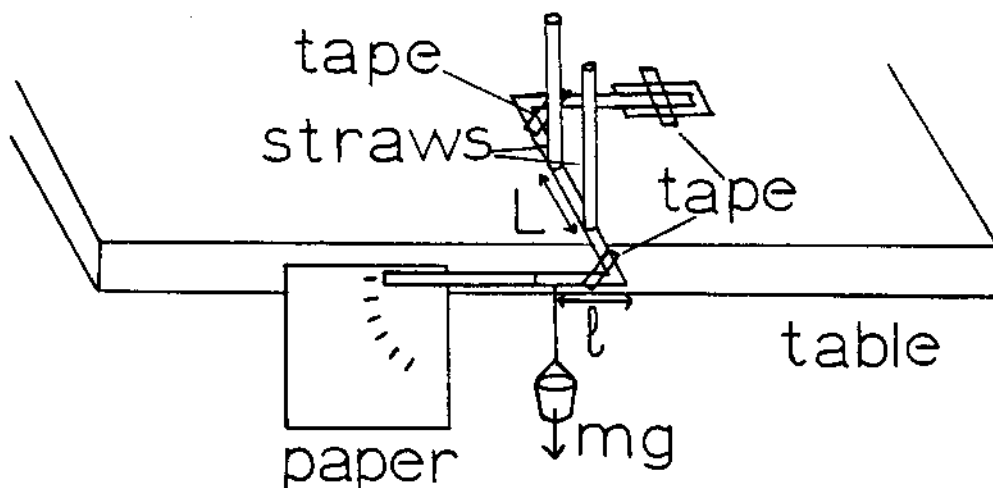


EXPERIMENT 1.07. The Modulus of Rigidity of a Drinking Straw.

Materials needed: Three straws, marbles, cup, string, sticky tape.

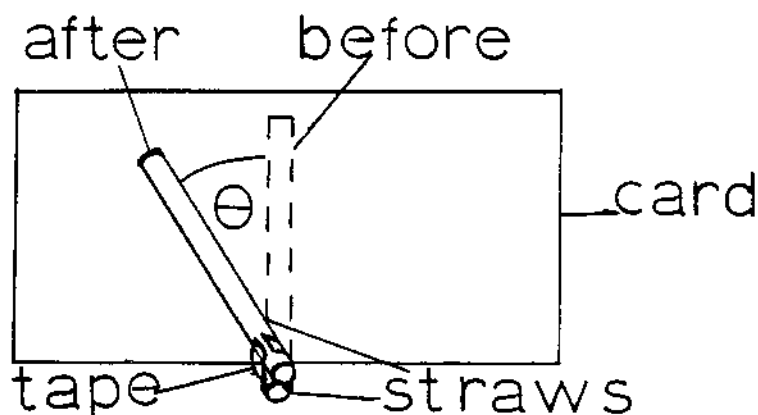
Procedure: We wish to measure the torque (moment of the force) which produces a given rotation of the drinking straw.

Attach the straw to the table, as in experiment 6, but now stick two other straws as shown, to measure the rotation over a fixed length, say 10 cm of the straw.



The mass of the marbles should be known. The information we need is:

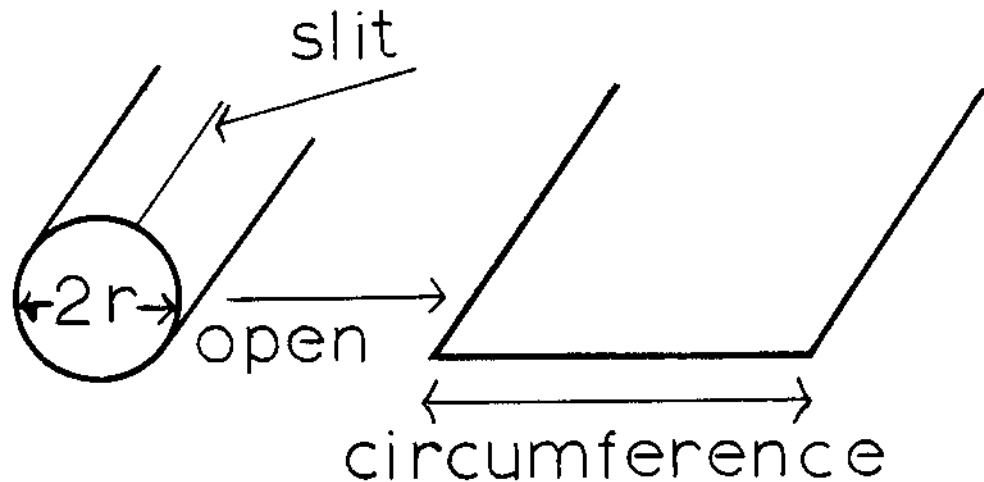
1. the mass of marbles in the cup m
2. C , the couple rotating the straw, $mg l$, where l is shown in the figure.
3. The angle of rotation of the more distant straw, θ_1 found by marking a card placed behind it, before and after adding weights to the cup. The torsion constant for the straw, k , is given by couple = $k \times$ angle of rotation.
4. The angle of rotation of the nearer straw, θ_2 , found similarly



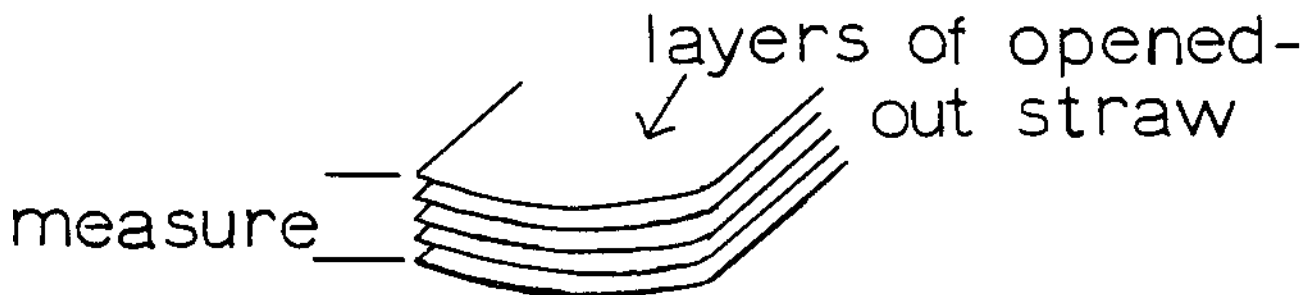
5. The length between the marker straws, L . The "torsional rigidity" can now be found which is defined as

$$\frac{CL}{\theta_2 - \theta_1}$$

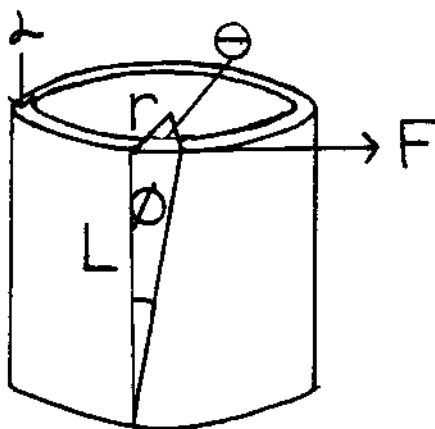
6. The diameter of the straw $2r$ must now be found, using a millimeter ruler. The straw can be slit, as shown, for more accurate measurement of the circumference, $2\pi r$.



7. The thickness of the straw material must be found, by superposing several layers, and using a millimeter ruler



Now the modulus of rigidity of the straw can be found.



The cross-sectional area of the straw is α , at a distance r from the axis of the straw, which is fixed at one end a distance L from α . The external couple C twists α through an angle θ . If the tangential force on α is F , the couple is given by

$$C = Fr$$

If the angle of shear is ϕ

$$r\theta = L\phi$$

The modulus of rigidity n is given by

$$\frac{\text{tangential stress}}{\text{angular deformation } \phi} = \frac{F/\alpha}{\phi} = \frac{CL}{r^2\theta\alpha}$$

Since the area of cross section of the straw is $2\pi r t$ where t is the thickness of the straw,

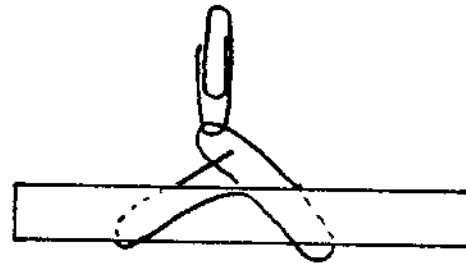
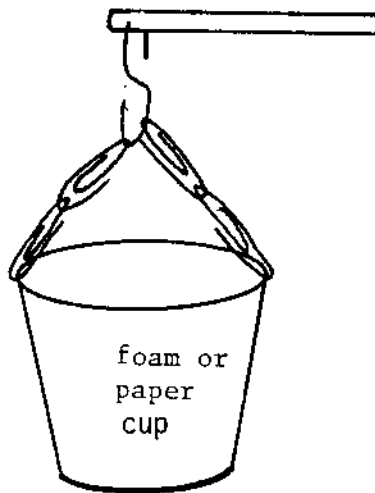
$$n = \frac{CL}{2\pi r^3 t} = \frac{mg\ell L}{2\pi r^3 t}$$

Hence the modulus of rigidity of polyethylene straws may be determined. The value should be approximately: 10^{10} dynes/cm².

Experiment 1.08. Equal armed balance

Materials needed: - Two paper cups, soda straw, marbles, paper clips.

Procedure: - Poke a hole through the soda straw at each end, and push a paper clip through, to support two cups as shown. Either poke a hole midway between the other two, or, in order not to reduce the strength of the straw, make a support as shown by bending a paper clip.



Now you have an equal-armed balance. Try weighing marbles against one another. Change the marbles from one cup to the other, keeping the number in each cup constant. What does it show?

Qualitative: Even with the same number of marbles, the balance is never exact, but you can tell approximately when balance occurs.

Quantitative: You can get a closer balance by adding paper clips to one cup or the other of the balance. How many paper clips balance one marble? When you have two marbles in each pan, how many additional paper clips are required to balance? Do different marbles weight the same? How big is the difference?

Experiment 1.09. Levers - the steelyard.

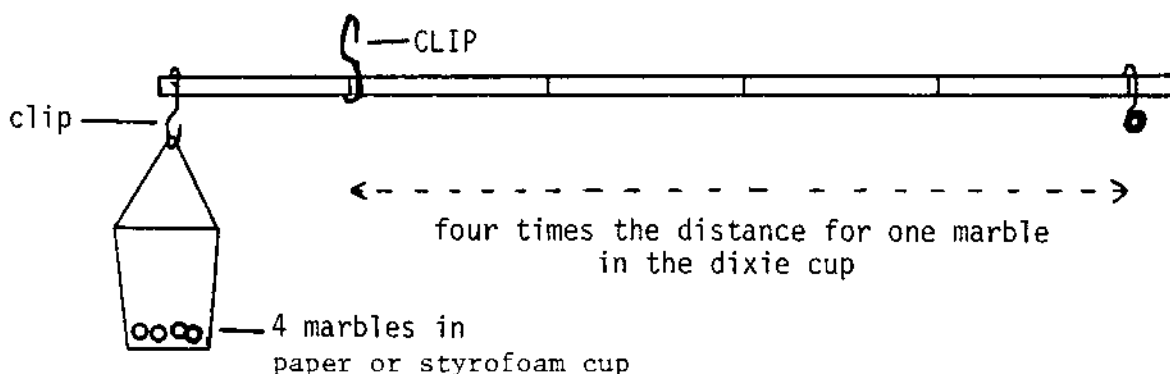
Materials - soda straw, cup, paper clips, string and marbles.

Procedure: - Poke two holes through the straw with the paper clips, one about 1/2 inch from the end, the other about 1 1/2 inches from the end. Attach a marble to a paper clip with sticky tape, or by bending the clip as shown. Now, support the cup by the paper clip close to the end, and slide the marble along the arm until it balances. Then add a marble to the cup, and balance again. Do this for several marbles and mark on the straw where the balance marble is using a marking pen or piece of sticky tape. You can use the steelyard to count marbles.

What does this show?

Qualitatively: - You can lift several marbles by one marble if you move it far enough away from the pivot or fulcrum. If you had a long enough (and strong enough) straw, you could lift the whole world with one marble.

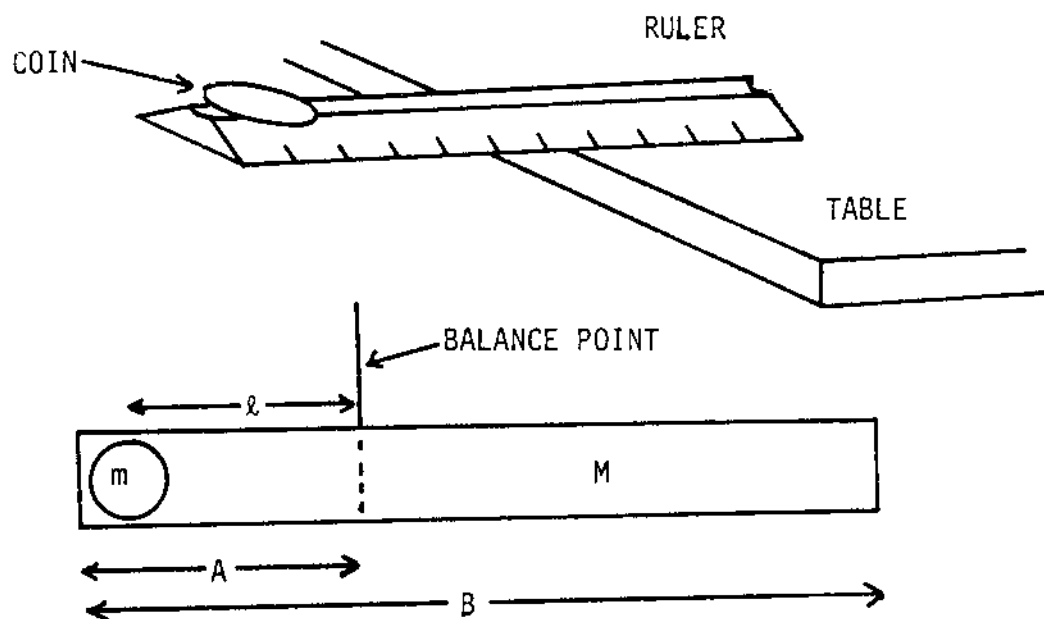
Quantitatively: - Notice that the distance you must move the balance marble every time you add another marble to the cup is always the same. This confirms the relation: -



force (1) x distance to axis = force (2) x distance to axis
for equilibrium.

The steelyard was used to weigh objects in the days of the Greeks and Romans. It is still used today for some accurate weighing machines.

A very simple balance may be made by placing a coin on the end of a ruler, and pushing it over the edge of the table until it overbalances.



If the mass of the ruler is M , and of the coin m , then taking moments about the balance point

$$mg\ell = Mg(B/2 - A)$$

Find the mass of the ruler from the masses of coins given in experiment 1.01.

Experiment 1.10 The letter balance

The letter balance is convenient for weighing small objects. Cut out the diagram shown, folding one side over the other, to provide more strength. Stick one penny on each side of the paper, in the marked spaces, if you wish to use the gram calibration, or two pennies on each side (four pennies in all) for the ounces. Push a paper clip through the hole marked W, and hang the letter to be weighed from it. Another paper clip provides the pivot P as shown in the diagram, and a straightened paper clip, B in the diagram, hangs vertically to provide the pointer.

The letter balance is calibrated by taking moments about the pivot P. The line joining the center of mass of the balance, complete with pennies and paper clip, to the pivot must be at 90° to the line joining the pivot to the point W from which the weight hangs.

If the distances are as shown in the diagram, at equilibrium

$$mgb \cos \theta = Mga \sin \theta$$

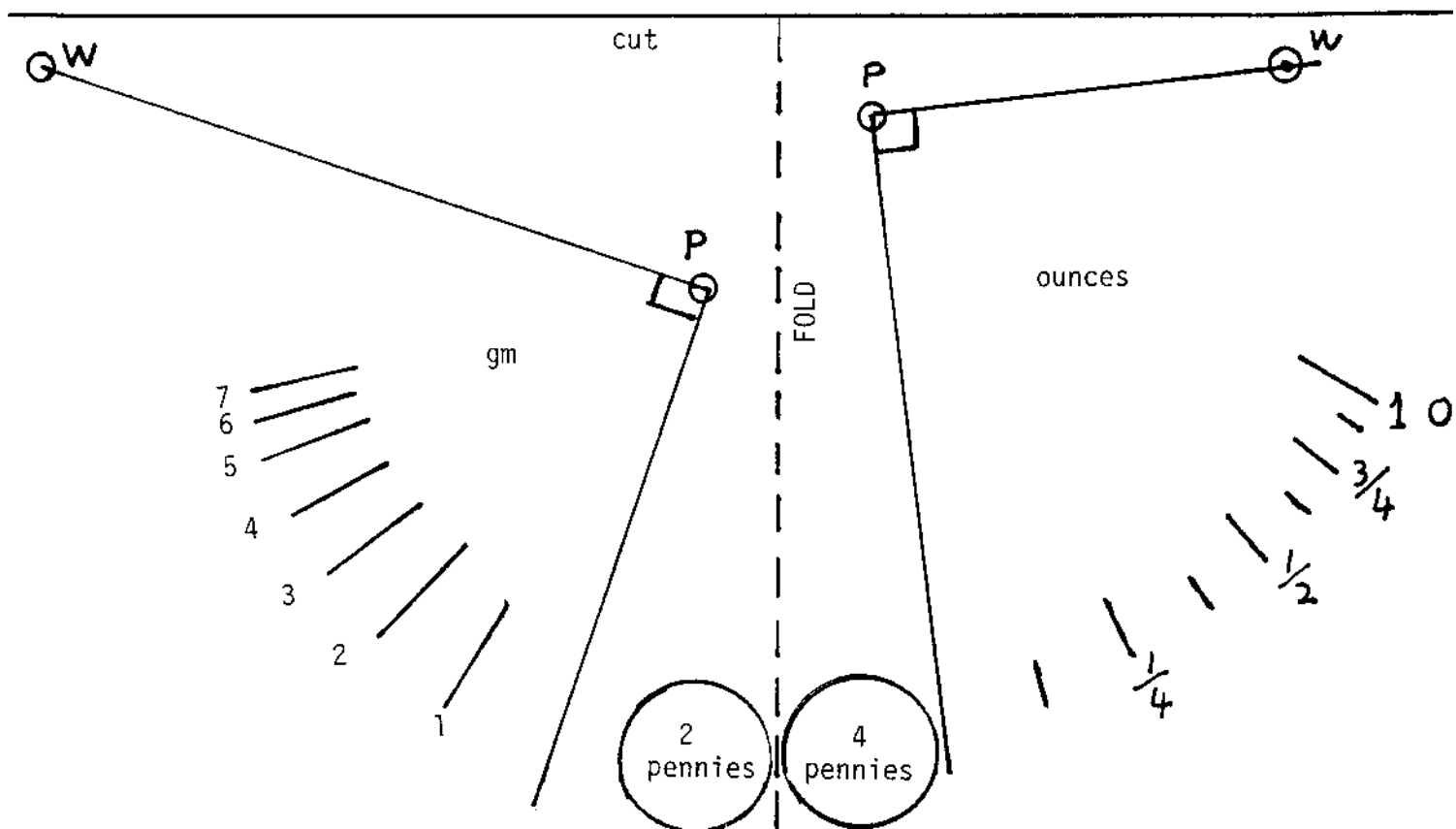
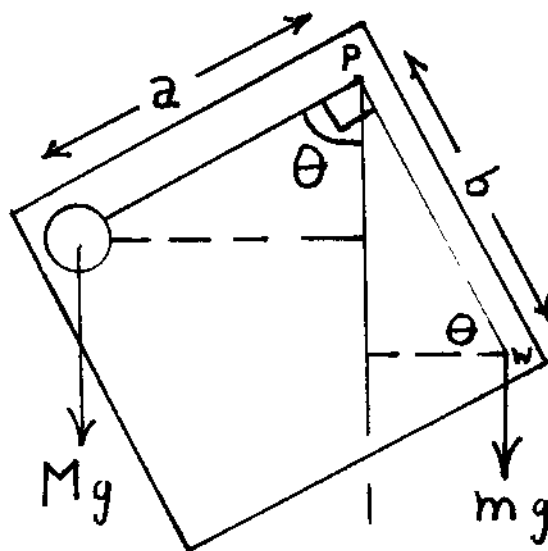
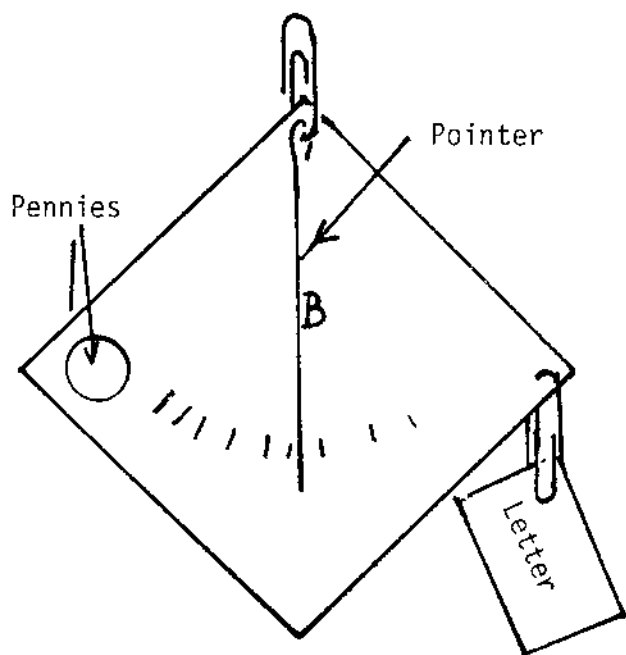
where M is the mass of the balance, and m is our unknown. Hence

$$m = \frac{Ma}{b} \tan \theta$$

Since a penny weighs 3 gms, the scale may be drawn directly on the card. This has been done for the two scales shown.

Stick three pennies on the balance in place of the two or four, and calibrate it yourself by hanging dollar bills (each weighing 1 gm) or other objects from the weighing clip, and making a mark where the pointer hangs with no mass, or one or two dollar bills.

Check whether the tangent of the angle you measure is proportional to the mass of the object being weighed.

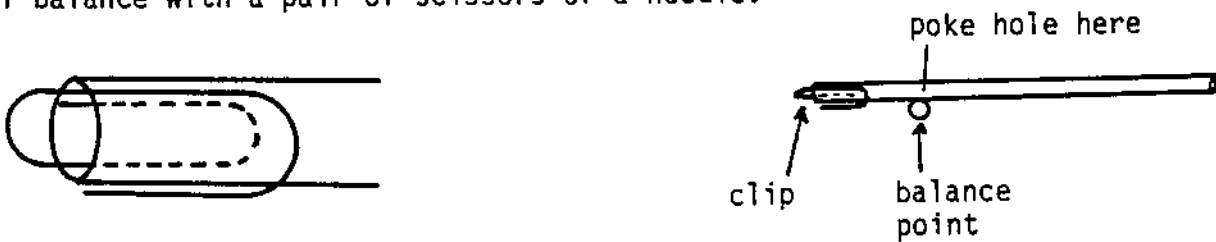


Experiment 1.11. The microbalance

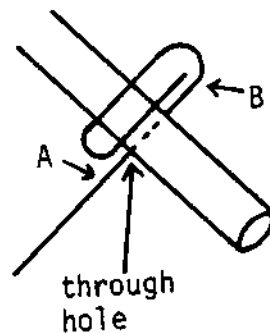
Materials: - Soda straw, paper clips, sticky tape, paper.

Procedure: - Only in sensitivity does this differ from the previous experiment. With this experiment, it is easily possible to weigh a human hair.

Take a soda straw and push the narrow part of two paper clips into one end. Balance it roughly on another straw, and poke a hole at the point of balance with a pair of scissors or a needle.

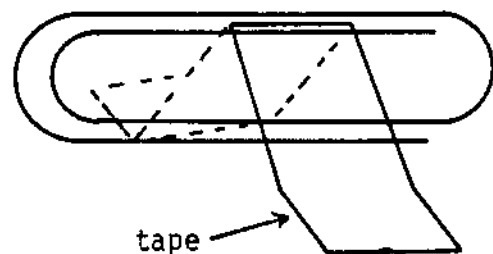
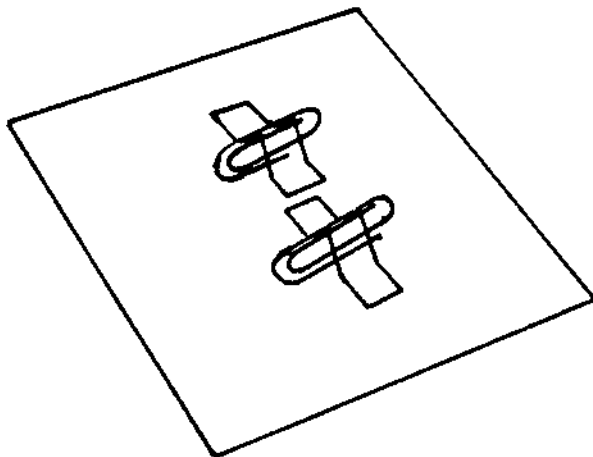


Unfold one end of a paper clip and push it through the hole, as shown.



Make sure the region from A to B remains perfectly straight.

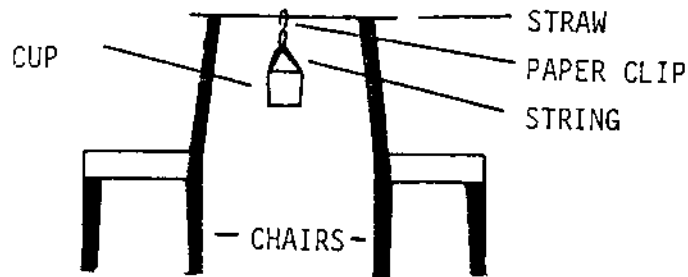
The balance must now be placed on frictionless bearings. The best we can do is to use two more paper clips. Tape **these**, as shown



Experiment 1.12. Stresses in beams

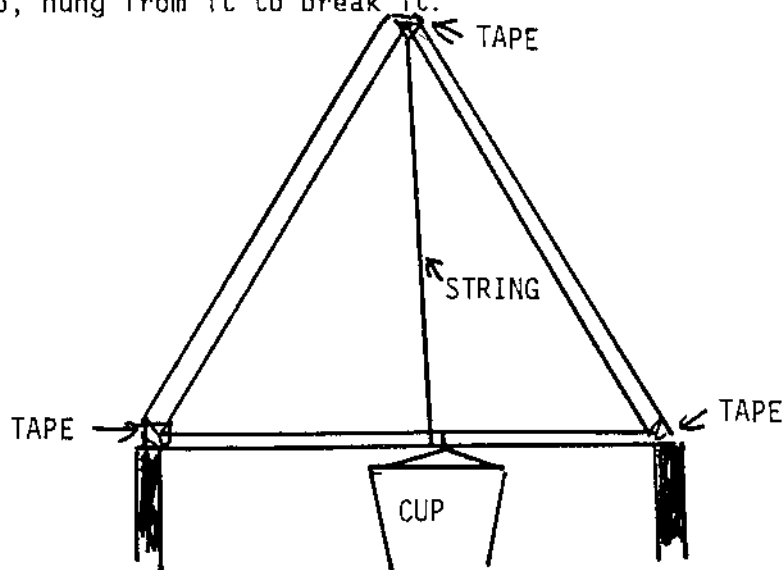
Materials needed - several plastic drinking straws, sticky tape, string marbles, foam or paper cup.

Procedure: Hang a cup from a straw by string and a paper clip as shown



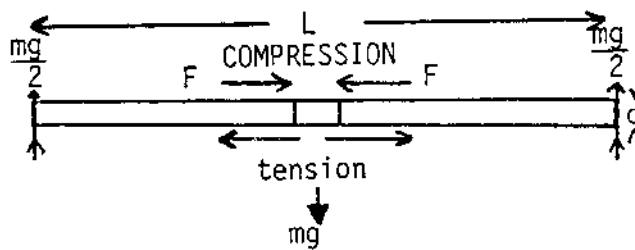
Support the straw as close to each end as possible. Add marbles to the cup hung from the middle, until it gives way. Do the same with the cup hung about a quarter from one end. Why do you think it needs more marbles close to the end? You will need a lot of marbles!

Now, take three straws, fasten the ends of two straws together, and fasten the outer ends using tape to the other straw, as shown. Tape a piece of string round the top, fastening it with tape to be sure, loop it round the lower straw, and again, see how many marbles break this girder. You probably won't have enough marbles, and it will take a book or two, hung from it to break it.



Qualitatively - a narrow girder, like the straw, breaks easily - a large girder distributes the force and can withstand a heavy load.

Quantitatively - the bending torque at the center is very large



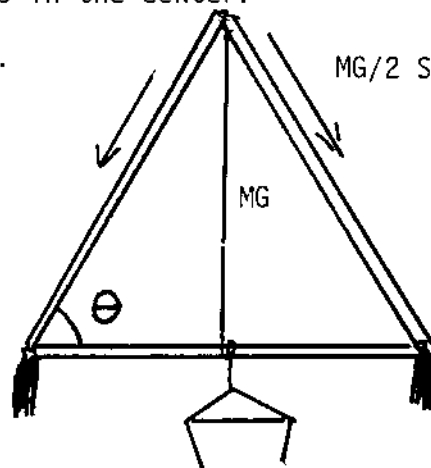
$$\text{torque} = \frac{mgL}{4}$$

$$\text{force at top } F = \frac{mgL}{4d}$$

With the weight a quarter from the end, the torque becomes $\frac{3mg}{4} \times \frac{L}{4}$

i.e. $\frac{3}{4}$ of what it is in the center.

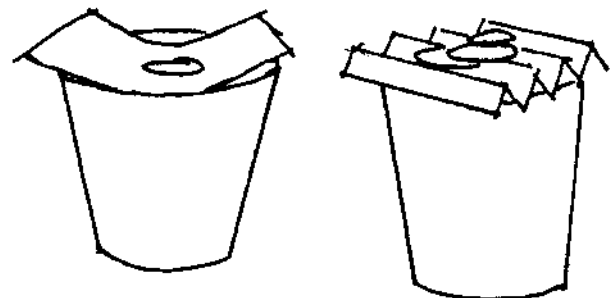
For the girder.



$$MG/2 \sin \theta = 0.6 MG \text{ if } \theta = 60^\circ$$

A simple example of the strength of corrugation.

Place a piece of paper the size of a dollar bill over a cup. A quarter placed on the paper will fall into the cup. If the paper is corrugated, however, with half inch pleats as shown, many quarters (or pennies) may be piled on without it breaking.



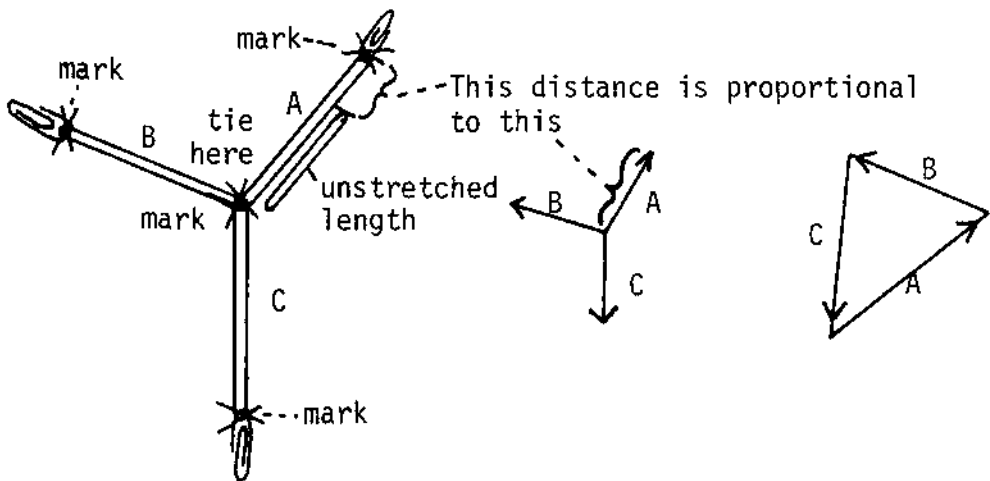
Experiment 1.13 The triangle of forces.

Materials needed: - three rubber bands (as much alike as possible), three paper clips, sheet of paper, pencil, string.

Procedure: Fasten the three rubber bands together as tightly as possible, as shown, using a piece of string. Now, hold two down on a sheet of paper, and stretch the third, so that they spread out under tension. Mark the point they are joined, and the ends of the rubber bands, on the sheet of paper.

Qualitative: The rubber bands are under tension, that is, they are exerting a force on one another, but they are in equilibrium - so the forces balance one another in order that there is no motion.

Quantitative - Subtract the unstretched length of each band from the stretched length.



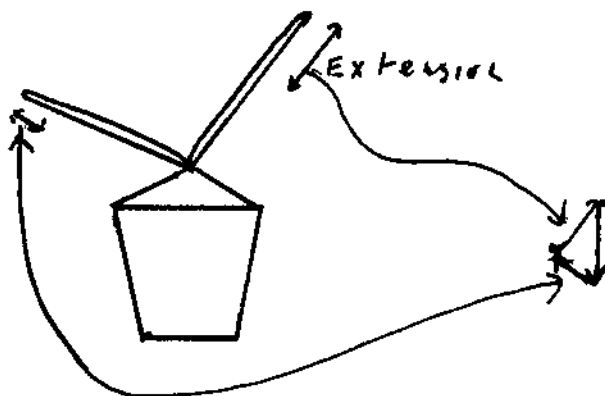
Make a diagram of the forces. You can represent each force by a length, and since the extension of each band is proportional to the force extending it, that length can be the extension itself. Now, the three forces, placed head to tail, should form a triangle, whose sides are parallel to the forces, as shown in the figure. This is the triangle of forces, determining the magnitude and direction of the forces for equilibrium.

Unfortunately, because it is rare to get three comparable rubber bands, about 20% agreement is all that can be expected of this experiment. However, you can use one band as standard. Leave one band loose, and extend the other by pulling it against the standard, as shown. The extension of the standard



for the extension of the other band in the experiment can then easily be found and used in the experiment.

Another interesting experiment is to hang a cup of marbles from two rubber bands, as shown.



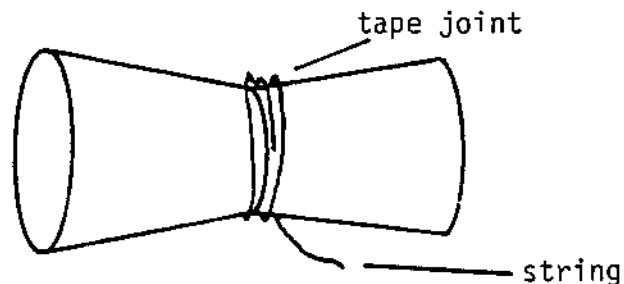
The resultant of the forces in the two rubber bands is found. The force of the marbles in the cup can be found by hanging from one band. Is this the same as the resultant? As the bands are stretched more and more, what happens to the cup?

Another problem with this experiment is that the rubber bands do not obey Hooke's law--hence large extensions should be avoided.

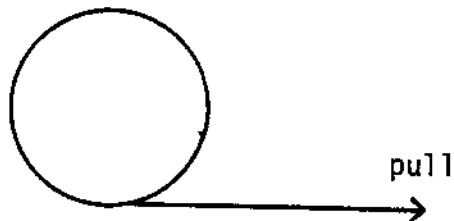
Experiment 1.14 Application of couples and torque.

Materials: - Two foam or paper cups, sticky tape, and string

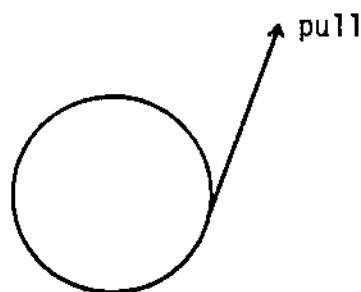
Instructions: Attach the bottoms of the cups together as shown.



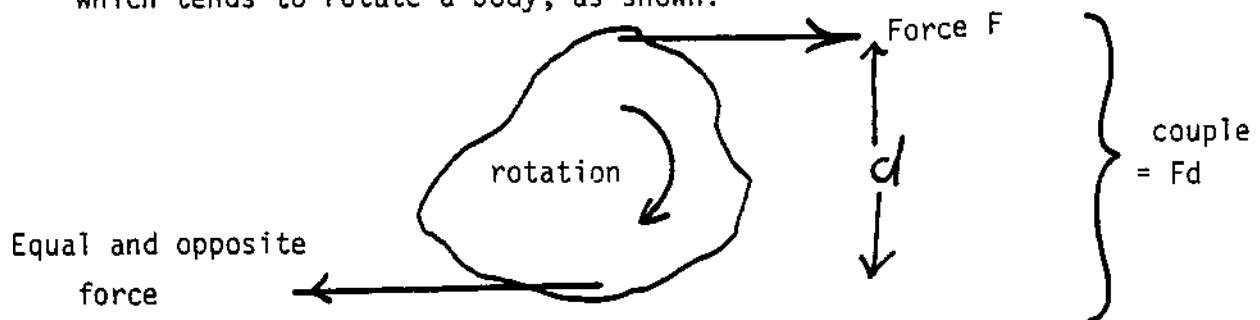
The string is taped to the joint and wrapped around it several times. Now, place the cups on the floor horizontally, hold the string level with the floor, and pull. Which way do the cups move?

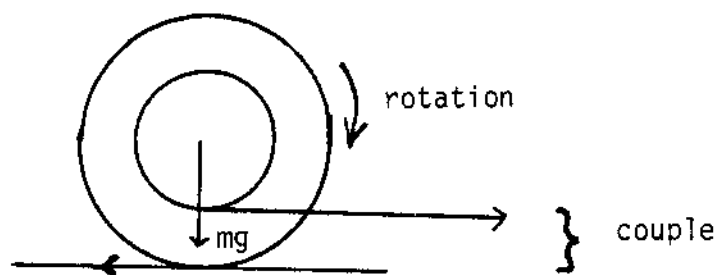


Hold the string at an angle of about 80° and again pull.

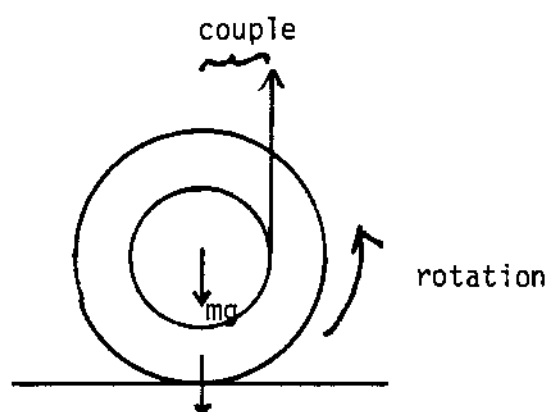


Quantitative: A couple consists of two parallel forces, equal and opposite, which tends to rotate a body, as shown.

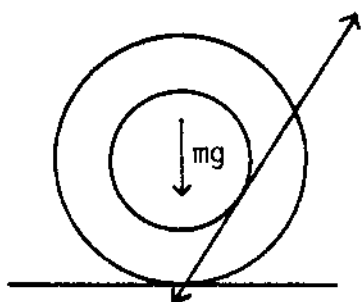




The couple acts to roll the cup toward the observer.



In the above position, the couple rotates the cup in the opposite sense.



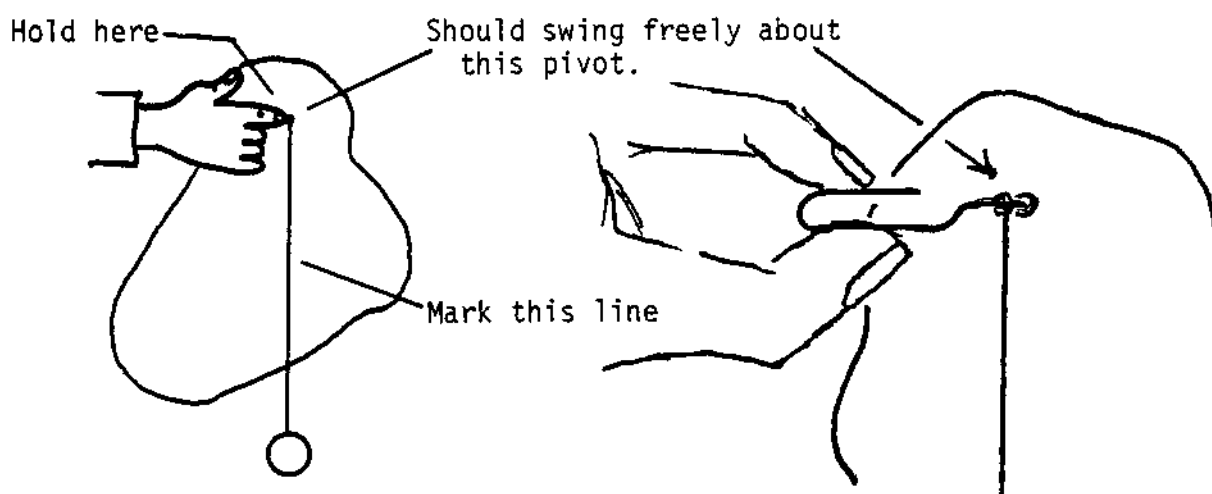
In the position shown, no rotation is possible because the direction of the string passes through the point of contact with the floor. The cups, when pulled, will skid, without rolling, along the floor.

Experiment 1.15. Center of mass (gravity)

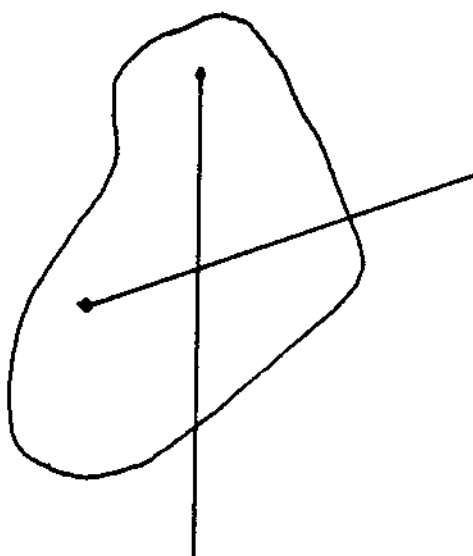
Materials: scissors, paper, pencil, string, paper clip, marble, cello tape

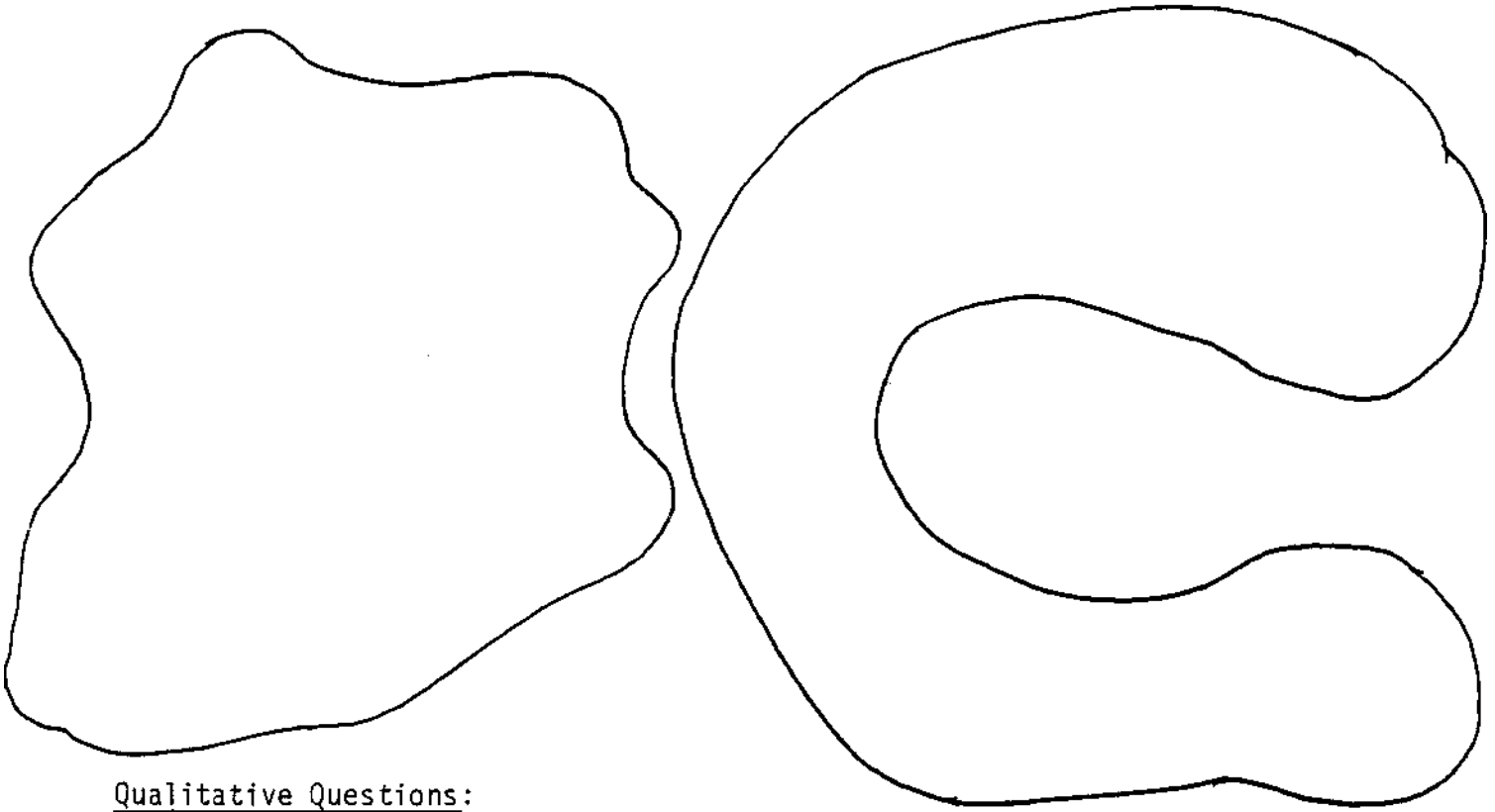
Procedure:

When an object is supported from a frictionless pivot, the center of mass lies directly below the pivot. Cut out the shapes over (and any others you would like. Open up a paper clip, tie one end of a piece of fine cord to the clip, and cello tape a marble to the other. Punch a hole, as shown, through one of the points in the cutout and hang it vertically. Mark where the string lies against the paper.



Repeat about another point of suspension. Where the lines cross is the center of mass. (C of M. or C of G.)



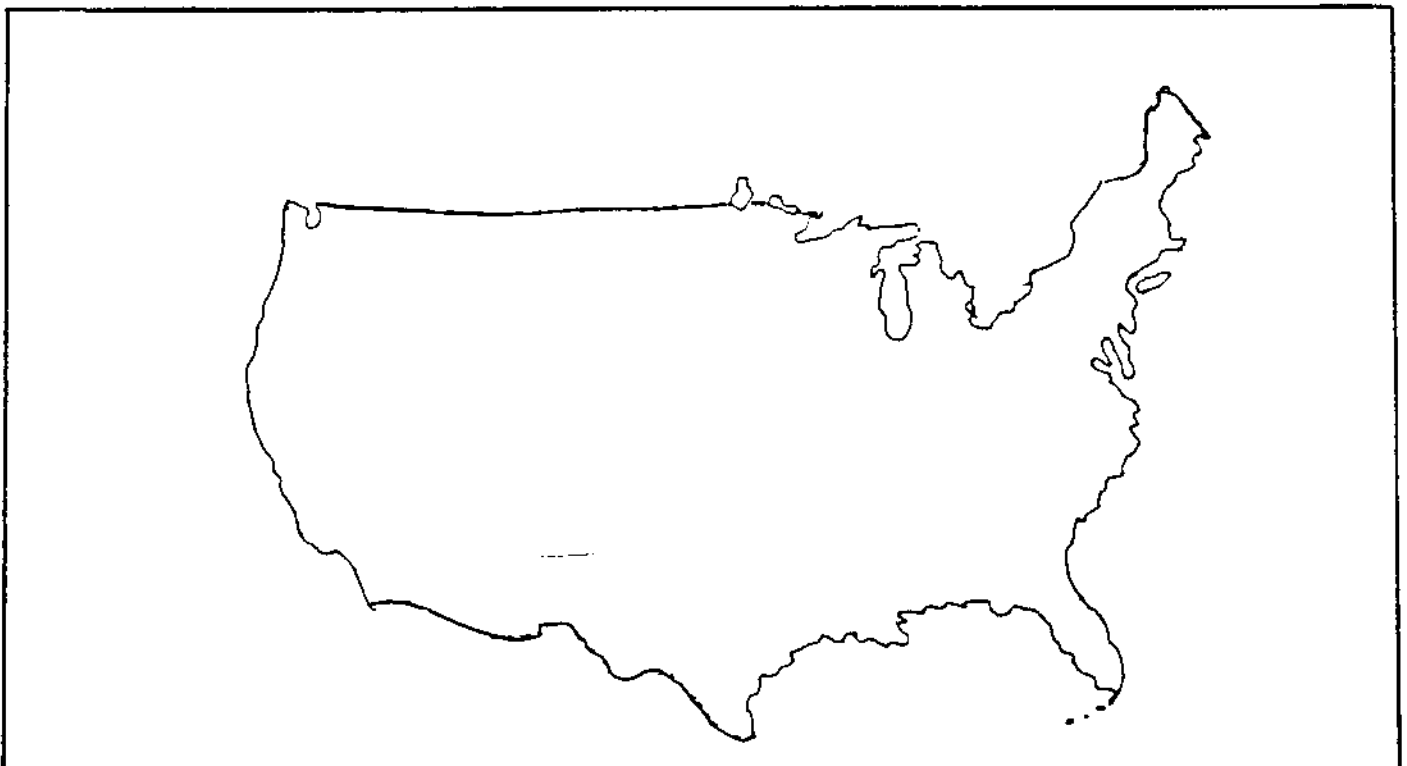


Qualitative Questions:

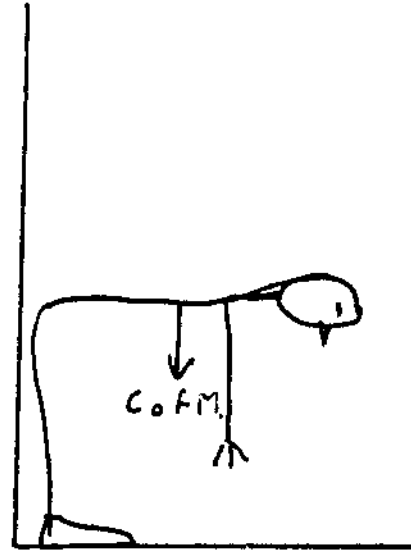
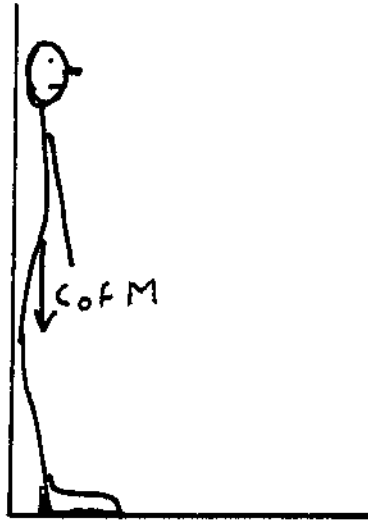
Object A: Why would you expect to find the center of mass near the middle?

Object B: Can the center of mass lie outside the object?

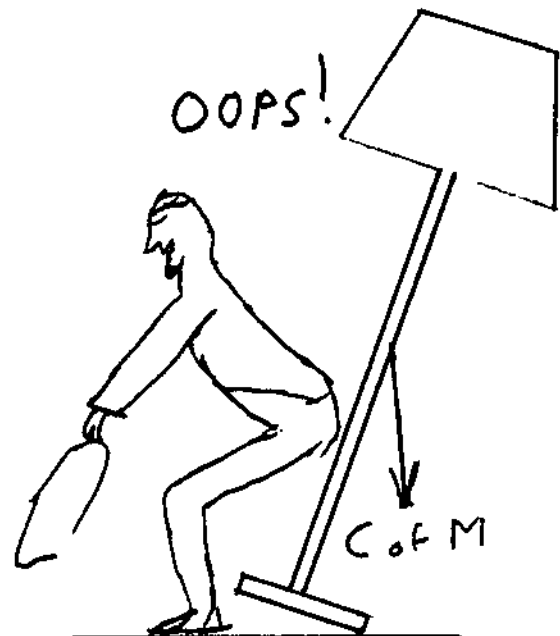
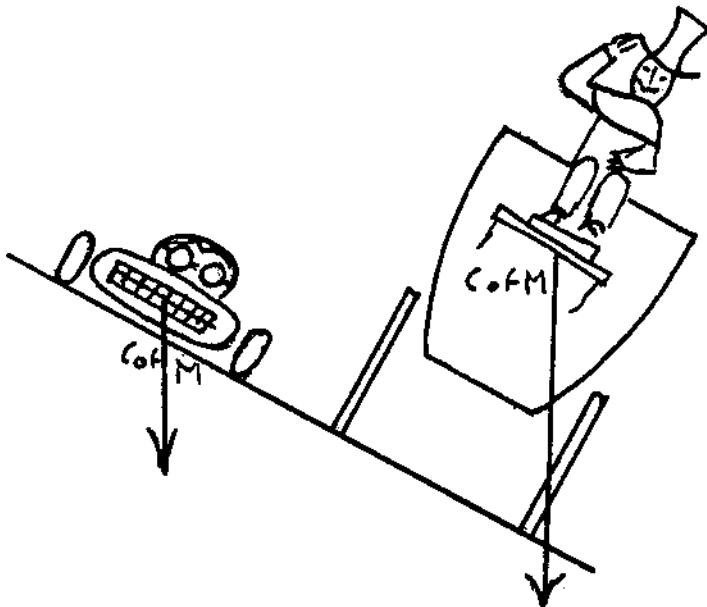
Quantitative Questions: Calculate where the center of mass of object C should be. Cut out C and check whether you are right. Then cut out the map of the US and C find out where its center of mass is.



Why is the center of mass of a body so important? Stand with your back to the wall and try to touch your toes. You fall over because your center of mass lies ahead of your toes.

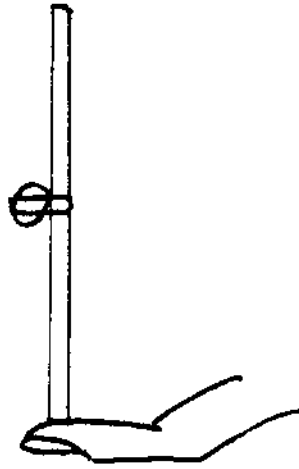


This is also why race cars have a low center of gravity so they won't turn over, whereas a stagecoach will.



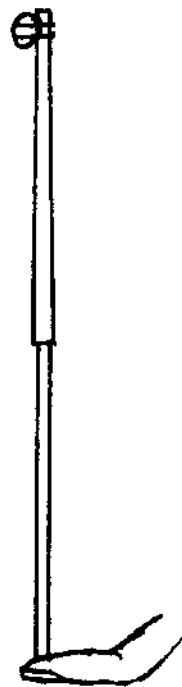
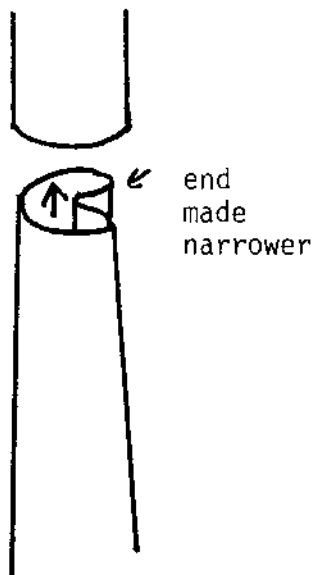
Or a tall lampstand is unstable.

An interesting experiment on balancing may be carried out using a marble and two straws. Fasten the marble half way up one straw, and try balancing it on one finger.



Now, push the end of one straw into the other (this is most easily done by removing the end of one straw by pushing it in with the thumbnail, as shown,) attach the marble at the end, and again try. It is much easier. Why?

The mass is essentially the same in both cases, but the second case is easier, because the moment of inertia about the finger is larger, so it takes longer to accelerate the marble, and you can move your fingers under the center of mass more easily.



Some interesting center of mass experiments can be performed using the human body¹, hence lending themselves to class participation (see Chapter 14).

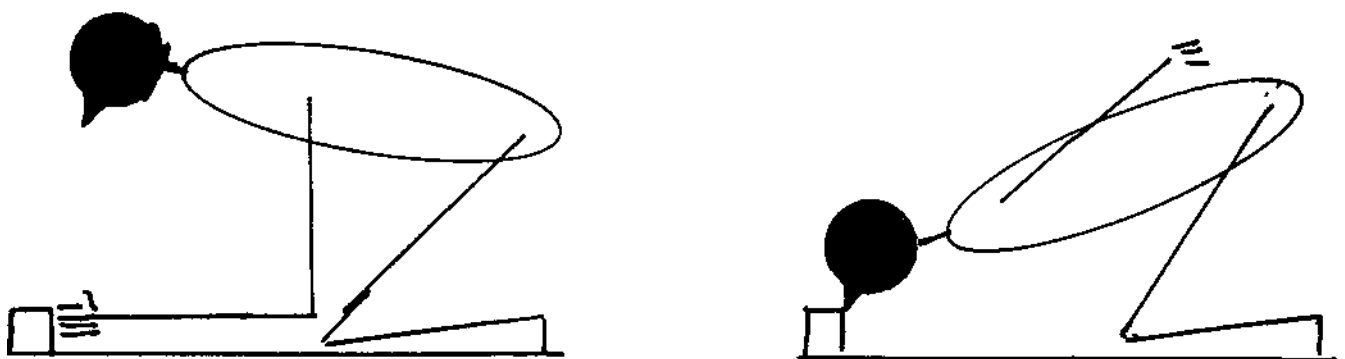
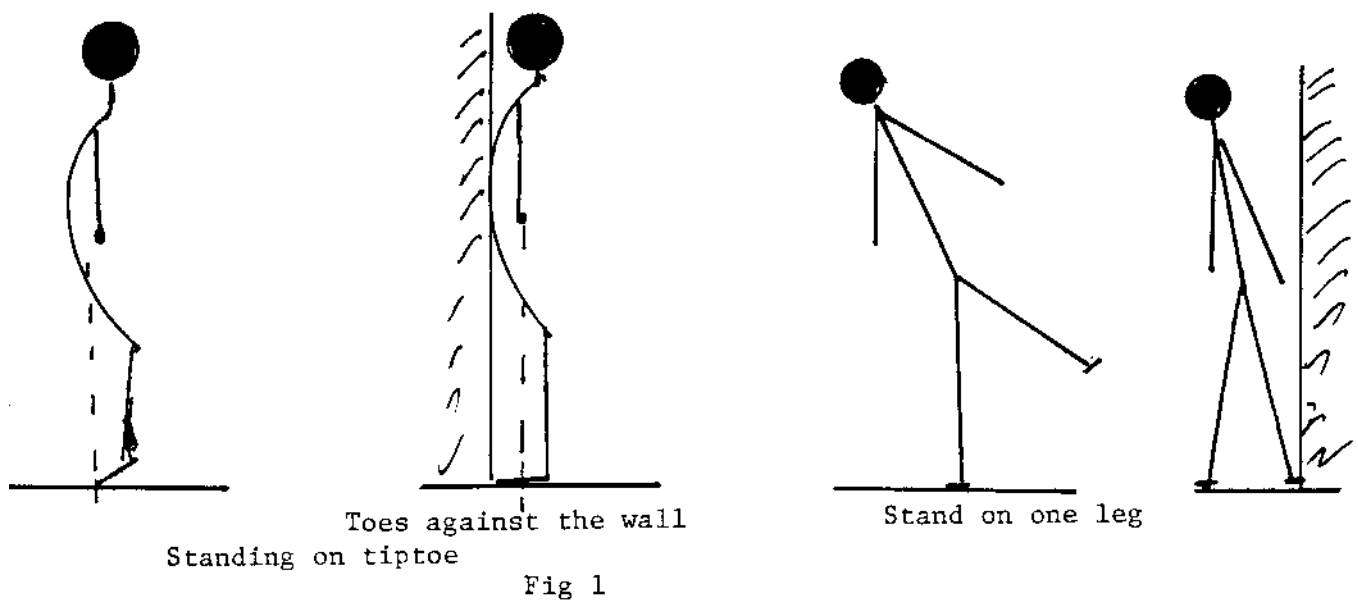
In the first demonstration (Fig. 1) a student is asked to stand on her toes normally, then with her toes against a wall. On her toes normally, the student automatically maintains the position of her center of mass directly above an axis of rotation through the toes. The torque acting on the student due to the force of gravity is then zero, and unstable rotational equilibrium results (the torque due to the force exerted of the student by the floor is, of course, zero too). When the student stands against the wall, it is impossible to bring the center of mass far enough forward so that it lies above the toes. Hence, rotational equilibrium cannot be achieved, i.e. a person cannot stand on tiptoes with the toes against the wall.

A similar experiment is shown in the same figure. In each case the student can maintain equilibrium standing away from the wall, by positioning his center of mass above a rotation axis through a foot, (or feet) but cannot do so when standing adjacent to the wall.

Another demonstration (sometimes used as a party trick)² shown in figure 2, demonstrates the difference in position of the center of mass of a female and male student. A kneeling student first places her elbow, arms and hands together (as if "praying") with the elbows touching the knees and the forearms along the floor. A matchbox or other object is placed at the students fingertips. The student then clasps her hands behind her back, and is instructed to knock the matchbox over with her nose without her entire body falling over. In general, female students can perform the task

whereas males cannot. Equilibrium in the kneeling position can be maintained as long as the center of mass does not move forward beyond a point above the knees, or backward beyond a point above the toes. Because the C of M of a male is closer to the head than a female, a typical male cannot knock over the matchbox without moving his center of mass forward of the knees, thereby tripping over.

1. Ernie McFarland, Physics Teacher 21, 42 (1983).
2. J. Watson and N.T. Watson Phys. Teach. 20, 235 (1982).



Experiment 1.16 Friction and the Climbing Monkey

Materials: Card, Tape, Straw, String

Akio Saitoh of Nara-Ken Japan provided this delightful experiment on friction which he calls "going up a tree." Figure 1 shows how to construct this apparatus.

1. Cut the sticky tape, attach it to the drinking straws, and fix them to the cardboard with an angle of about 20° between them.

2. Pass the string through the two straws, and hang the string on the nail as shown in Fig. 2.

3. Hold both ends of the string taut, pull on each end of the string alternately, and the cardboard will climb the string. You can draw a picture of a monkey as shown in Fig. 3 on the other side of the cardboard, and it looks quite amusing to see the monkey climbing.

When we pull on the left hand string, as shown in Fig. 4, the cardboard tilts to the left and the force normal to the straw on the right hand side, N_R , is larger than that on the left hand side N_L (Fig. 5). The left hand straw can slide up along the string, whereas that on the right cannot because the maximum force of static friction is proportional to the normal force.

What is the minimum tension in the string so that the monkey just begins to climb? If we assume that the string passes through the left hand straw without friction because the card is tilted, then the weight of the card is supported by friction with the right hand straw, so that $mg/2N_R = \mu$ where m is the mass of the card and μ is the coefficient of friction. But $N_R/T = \sin \theta$ where θ is the angle between the straws, if we assume the left hand straw in line with the string. So

$$T = \frac{mg}{2\mu \sin \theta}$$

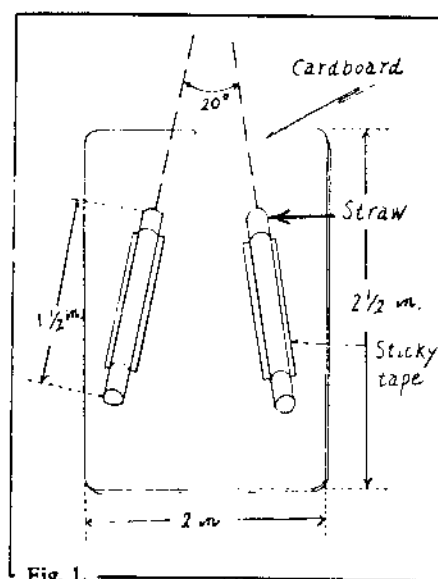
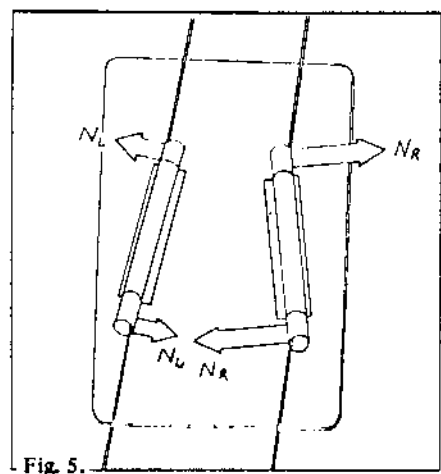
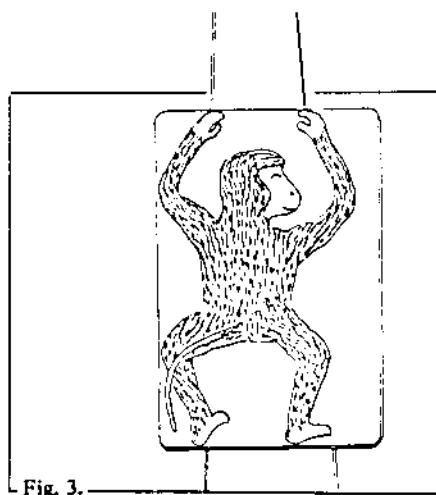
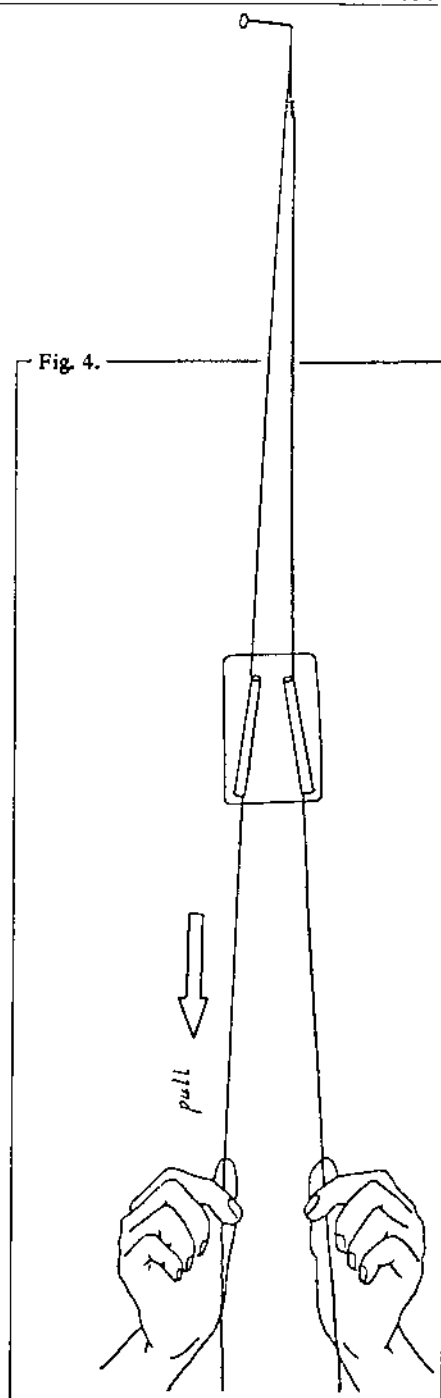
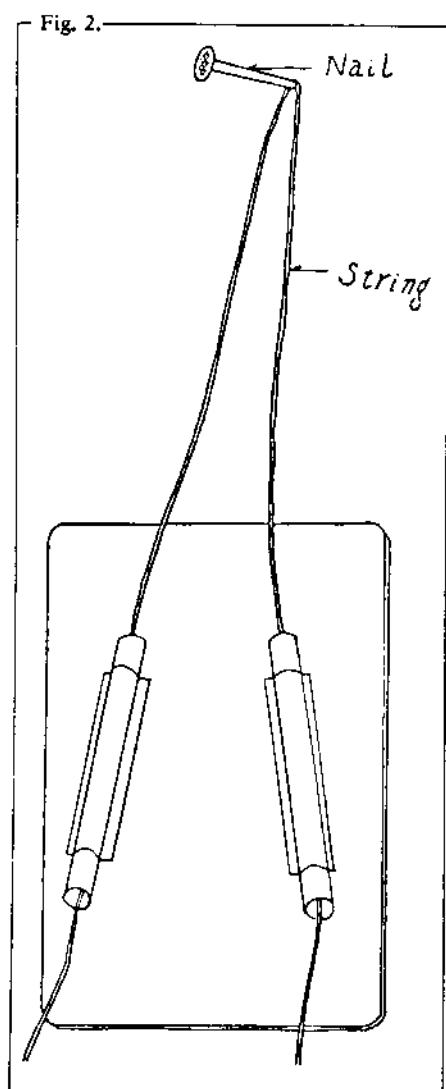
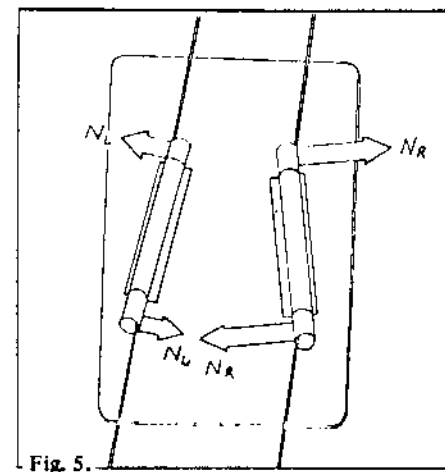
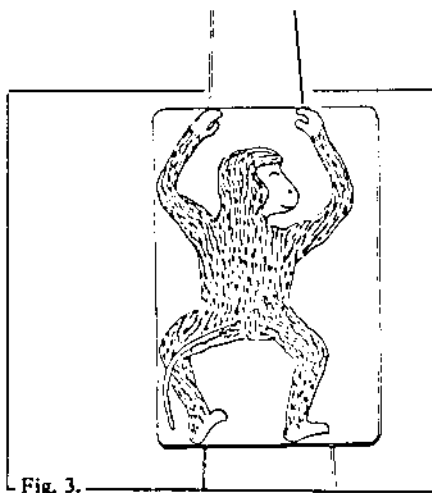
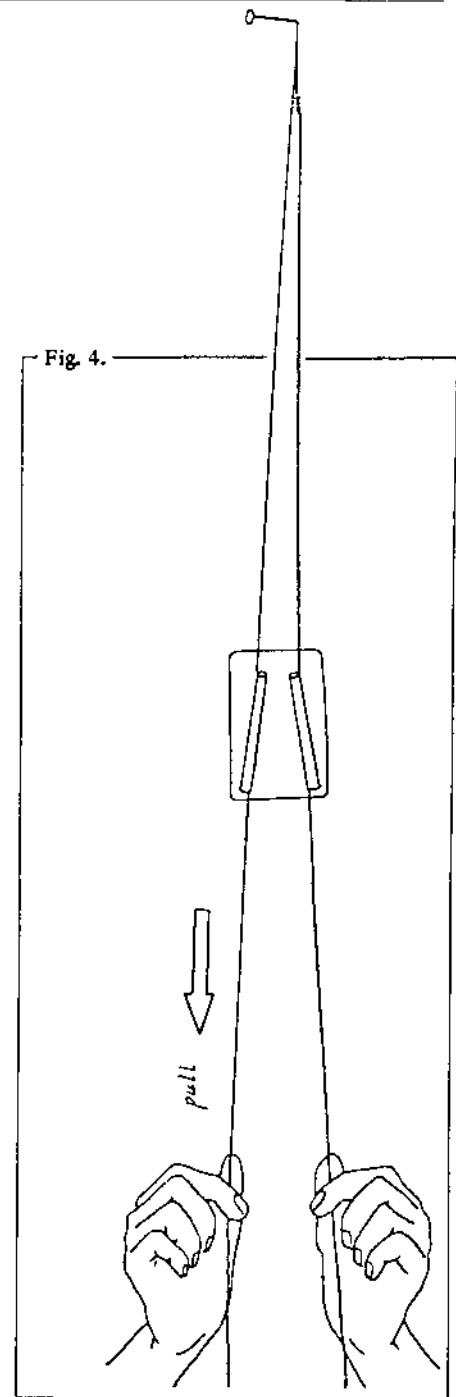
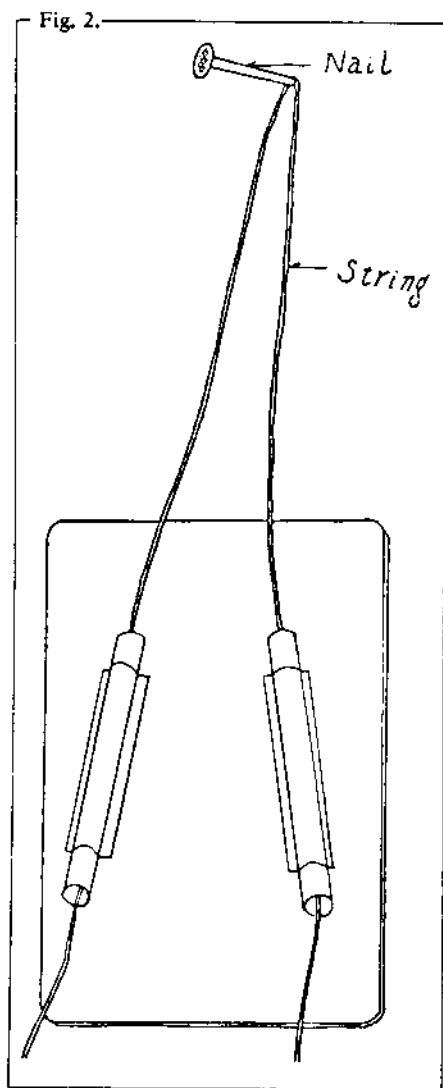


Fig. 1.

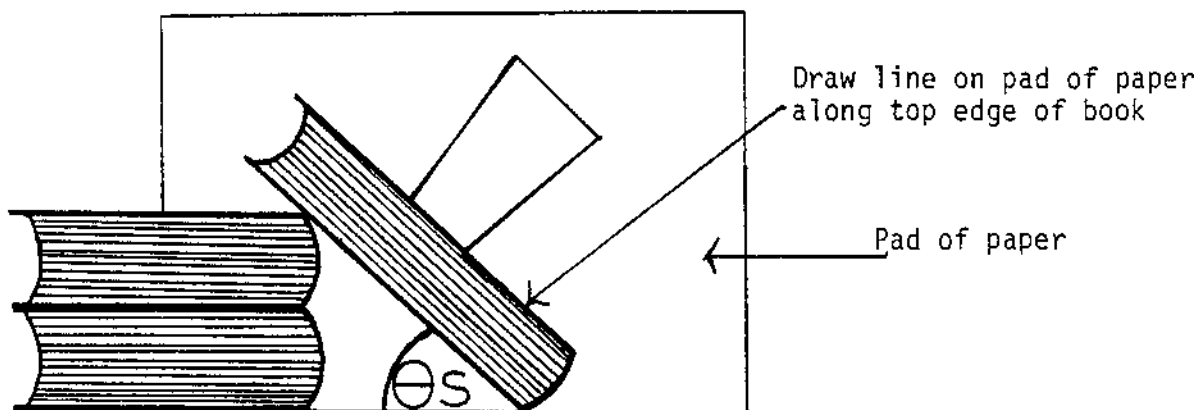




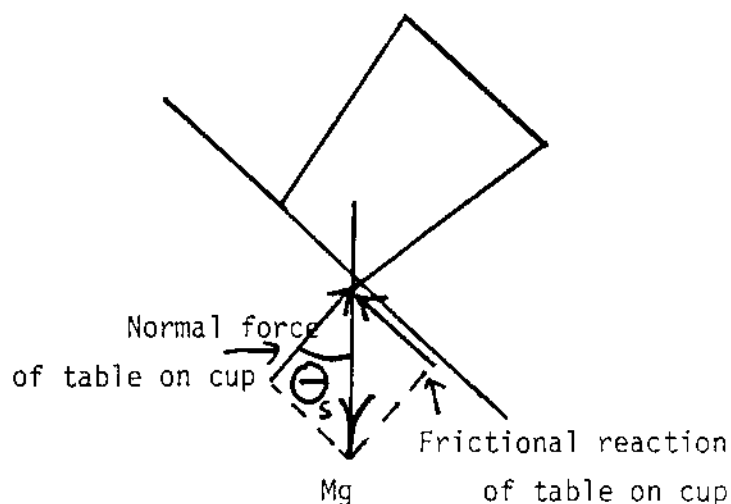
Experiment 1.17 Friction

Materials: cup, paper, marbles, books.

Procedure: Place four marbles in the styrofoam or Dixie cup. Put the cup on a book, and gradually tilt the book until the cup slides down it.



Repeat this several times, drawing a line along the top edge of the book placed against a pad of paper as shown each time. You can now measure the angle of tilt at which the cup starts to slide θ_s using the protractor shown at the beginning of this book. The tangent of this angle is the coefficient of static friction. This is the ratio of the force causing the cup to start sliding to the force holding it down on the surface as shown.



Now put a lot of marbles in the cup, and again measure the coefficient of static friction. Answer the following questions:

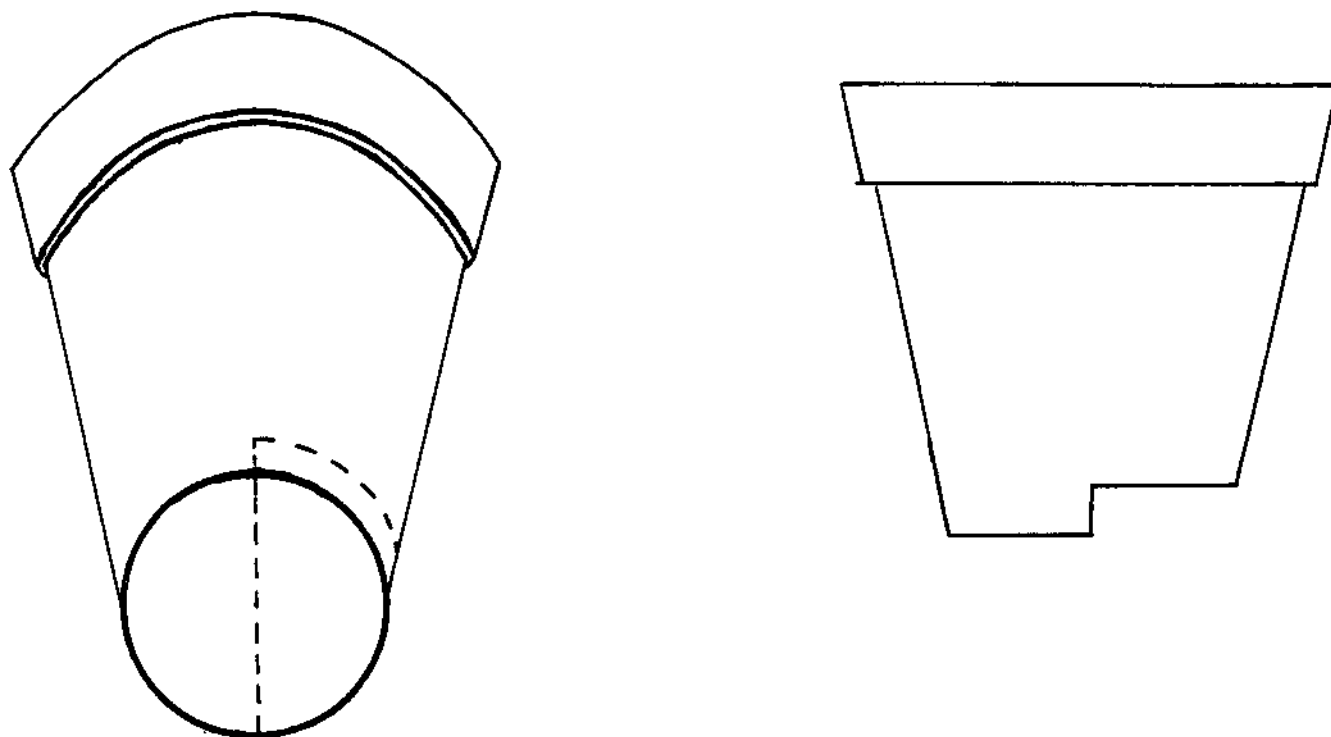
1. Do the measurements, taken under similar conditions, always give the same value for the coefficient of friction, or is there a spread. How wide is this spread?

2. Is the coefficient of static friction independent of the force holding the cup to the book?

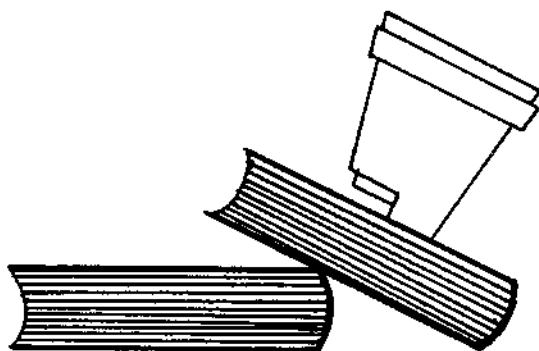
There is generally a spread of measurements, because neither the book nor the bottom of the cup are very uniform. The value of μ_s is approximately independent of the number of marbles in the cup, at least over a fairly wide range, showing that it is the nature of the surfaces which counts. Now repeat the experiment using a different book - try to use one book with a rough cover, and one with a smooth cover. Does the value of μ_s change appreciably? It is generally large for rough surfaces, small for smooth ones.

Set the book at the angle for μ_s and try replacing the cup - it is very difficult to do, showing that, once the cup starts to move at all at this angle it will continue to slide. Reduce the angle of tilt of the book until, when given a slight push, the cup does not continue to slide on down the book. The tangent of this angle is the coefficient of sliding friction, μ_k . It is always less than μ_s , since it takes less force to keep the cup sliding than it does to start it. Is μ_k independent of the number of marbles in the cup? Again, this is approximately true.

With a pair of scissors, cut away half the bottom of the cup, as shown



Place a second cup inside the first, put ten marbles in, and again measure the coefficient of friction.



Is it the same as for the whole cup? Only half the area of the bottom of the cup is now in contact with the book. If the coefficient of friction is the same, it follows that it depends only on the nature of the surfaces in contact, not on their area.

Experiment 1.18- Relative density

Materials needed: - the torsion balance (or one of the other balances you have made), marbles, foam or paper cup.

Procedure: - Read the balance as you add from one to ten marbles. Now, cut a V notch in a cup, and fill it with water until the water runs out of the notch. Hold it over the empty cup on the balance, and carefully drop marbles into the water, one by one, so that it overflows into the balance cup. The water in the balance cup, now has the same volume as the ten marbles.

Qualitative - note that the water does not twist the straw as much as the marbles did. The water weighs less than the marbles, and we say it is less dense.

Quantitative - Relative density is defined as

$$\frac{\text{Weight of object}}{\text{Weight of a equal volume of water}}$$

So, the ratio of the scale reading for the marbles to that for the water is the relative density of the marbles, which is numerically the same as the density, (mass per unit volume) since water has a density 1. You can measure the density of other objects, coins etc. the same way.

Note: One of the other balances can be used in place of the torsion balance, provided you weigh the marbles against the mass of water they displace.

The density of common glass varies from 2.1 to 2.8 gm/cc.

The Hydrometer^{*}

Seal one end of a straw with tape so that it is watertight. Attach a marble to the bottom of the straw with sticky tape so that it stands upright in the cup of water, floating about half submerged, as shown. Mark the straw at the water level with a pen. (If you have a tall cup or jar, it will let the straw float deeper).

Place the straw in a cup full of a second liquid of known density or specific gravity. The relative density or specific gravity (they are the same thing) is the ratio of the density of the material to that of water, and hence is a pure number. The density will have the same numerical value if the density of water is 1.0 which it approximates in the c.g.s. system, but it will be in units such as gm/cm^3 , (the S.I. system uses Kg/m^3 , for which the density of water is $1.0 \times 10^3 \text{ Kg/m}^3$). Mark the level. Some examples of suitable liquids are glycerine (relative density 1.26) kerosine (~ 0.8), gasoline (care! - 0.68 - 0.72) or solutions of common salt (10% by weight, 1.07; 20%, 1.148) or sugar (10%, 1.038; 20%, 1.081; 30%, 1.270).

Knowing the density of water (1.00) and of the other liquid, you can now calibrate the straw to produce a hydrometer which can be used to measure liquids of unknown density.

The most common uses of a hydrometer are to measure the densities of battery acid or sugar solutions: knowing the density of an aqueous sugar solution, one can calculate the alcoholic content when the sugar is converted into alcohol by yeast. The alcoholic content is not measured directly by density because the specific gravity of the alcohol solution

^{*} suggested by Michael Davis of North Georgia College, Dahlonege, Georgia.

(wine or beer) is too close 1. Thus at 20°C, the density of pure alcohol is 0.79 gm/cm³. If the relative density of the sugar solution is 1.1, the potential alcohol is 12%, which would have a specific gravity of 0.9749. For 1.15 it is 17.2%, corresponding to a relative density of 0.964.

Batteries contain sulphuric acid. A 10% solution has a relative density of 1.0661, 20% 1.1394, and 50%, 1.21885.

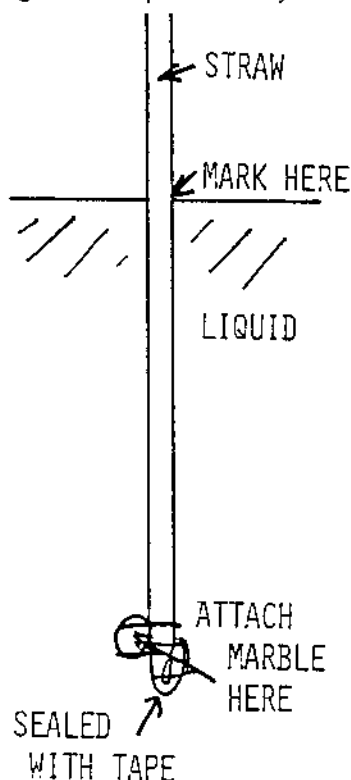
If the volume of the hydrometer under the water (density 1 gm/cc) is V , and it is found that placing the straw in a vessel of liquid having a new density ρ causes the straw to rise a distance d (increase in density) the change in volume of the hydrometer is $-\pi r^2 d$, where r is the radius of the straw, and

$$\rho(V - \pi r^2 d) = m = V,$$

where m is the mass of the hydrometer. Hence, increasing V also increases the sensitivity of the instrument, for

$$d = \frac{V(\rho - 1)}{\pi r^2}$$

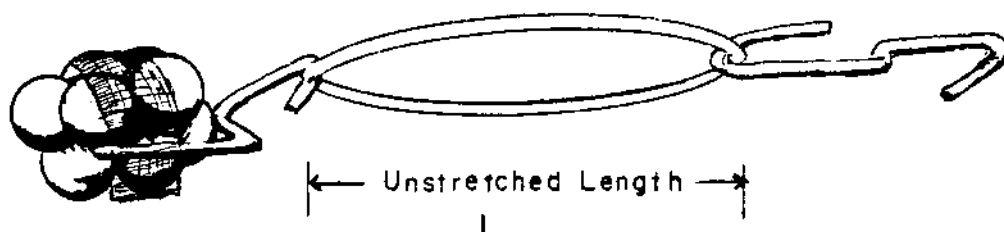
Attaching something of density near 1, (e.g. a lump of wax) to the hydrometer will increase V , and the sensitivity.



Experiment 1.19 . Buoyancy

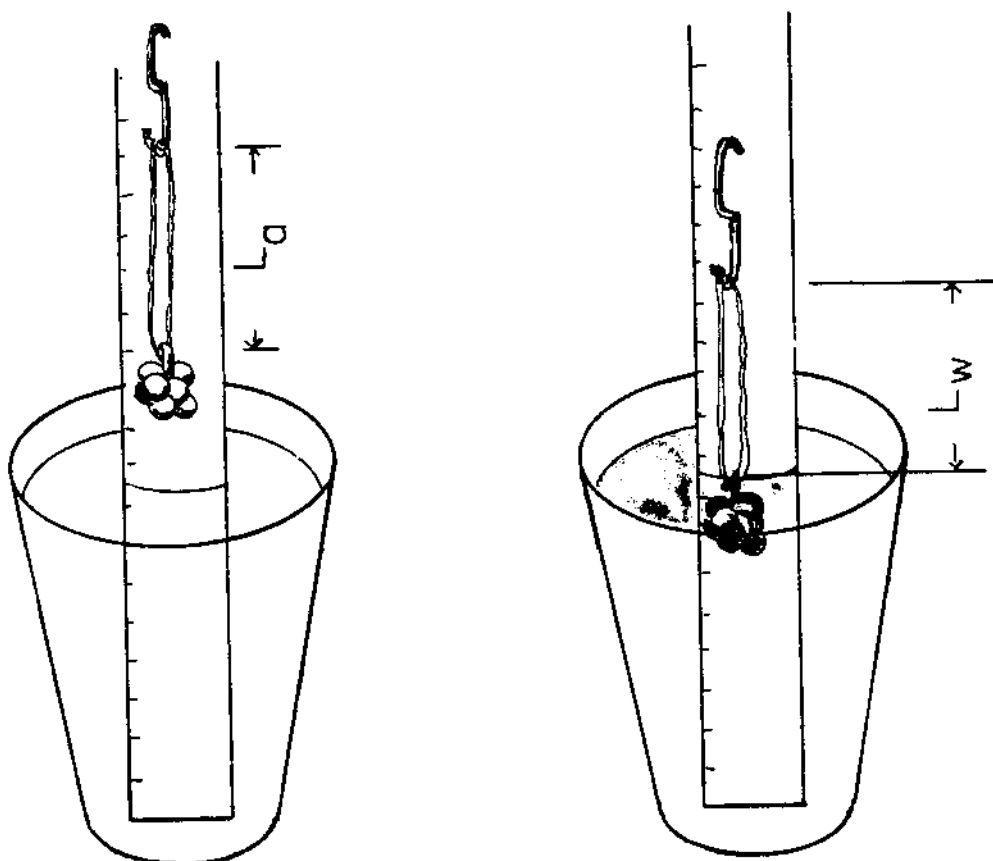
Materials: Cup, metric ruler, 2 paper clips, 8 marbles, stickytape, 2 rubber bands, water.

Procedure: Attach a rubber band to a paper clip that has been opened out into an L shape. Secure 8 marbles to the paper clip base with tape, leaving the rubber band free. (Without the paper clip base the marbles will slip off when wet.)



Open the second paper clip to form a hook for the top of the rubber band. Place the ruler in the cup straight up. Put about 7 cm of water in the cup. Measure and record (in mm) the unstretched length of the rubber band and the length of the band stretched while held out of the water, with the marbles on it. Lower the marbles into the water so that the bottom of the rubber band just reaches the surface of the water. Again measure the length of the stretched rubber band and record (in mm).

Qualitative: You will notice that the rubber band stretches less when the marbles are in the water. This is because the marbles are pushed upward by a force equal to the weight of the water they displace. This upward force is buoyancy that makes objects appear to lose weight in water.



Thought question: If you snip off a small piece of the cup and put it on the water, why does it float?

Quantitative: The relative density of any object is $\frac{\text{Mass of object}}{\text{Mass of equal volume of water}}$

In our case, the mass of the object is proportional to the amount the band stretches in air, d_1 , and the mass of the equal volume of water is proportional to the difference between this and the distance it stretches with the object underwater, d_2 , so the relative density =

$$\frac{d_1}{d_2 - d_1}$$

For example, the unstretched rubber band was 62 mm in one case, 75 mm with the marbles on, and 69 mm with the marbles in the water, so

$$\text{relative density} = \frac{13}{6} = 2.17$$

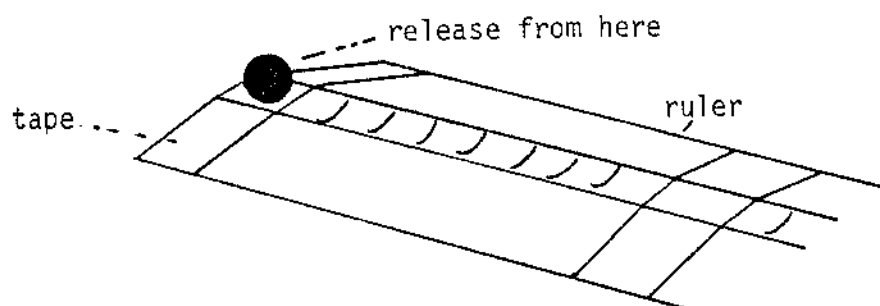
Length in air L_a		$L_a - L = d_1$	
Length in water L_w		$L_w - L = d_2$	
Unstretched length L		$\frac{d_1}{d_2 - d_1} =$	

Experiment 1.20. Velocity

Materials needed: - ruler, one marble, sticky tape.

Procedure: - Stick the half inch wide tape across the groove in the ruler at zero inches, 3 inches, 6 inches, and 9 inches. Tilt the ruler so that, when released from the top cellotape, the marble just makes it to the bottom. Now set it going, and listen to the clicks as the marble drops off each piece of tape and strikes the ruler - the clicks are always equally spaced in time, however fast you set the marble going with your finger. What does this show?

Try moving one of the pieces of tape. Are the times between clicks the same?



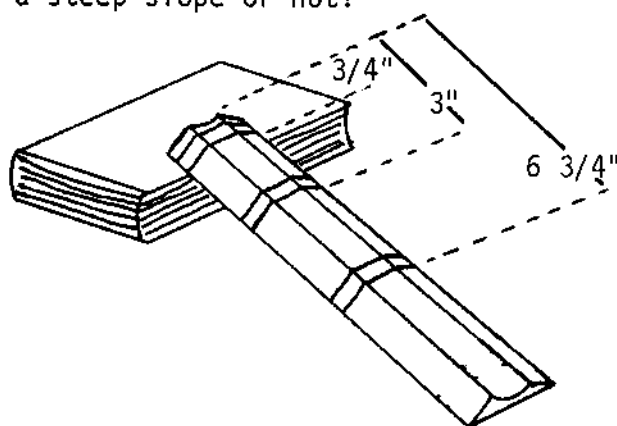
No horizontal forces act on the marble, which covers equal distance in equal times - this is called constant velocity. As we make the marble to go faster, the clicks are closer together, so higher speed, or velocity means the same distance in a shorter time, or a longer distance in the same time.

How fast does the marble travel? A crude estimate can be obtained by dropping a marble off the table at the same time you start rolling the one on the ruler. A normal table is 2 ft 6 in (76 cm) from the floor, so the marble takes 0.4 secs to drop. If the click on the ruler and that on the floor occur at the same time, the speed is the distance on the ruler divided by 0.4 in cm (or inches) per second.

Experiment 1.21 Acceleration

Materials needed: ruler, one marble, sticky tape

Procedure: Stick the tape across the groove in the ruler at zero inches, $\frac{3}{4}$ inch, 3 inches, and $6\frac{3}{4}$ inches. Tilt the ruler by two or three books. Now release the marble from rest at the top, so that it rolls rapidly down, making an audible click as it passes over each piece of tape, and at the end as it rolls off the ruler. Listen to see if the clicks are evenly spaced. Does it make any difference whether the ruler has a steep slope or not?



Qualitatively - as the marble travels down the ruler, it is subject to gravitational forces, making it cover larger distances (between pieces of tape) in the same time - so its speed (distance divided by time) is increasing - this is what we mean by acceleration under the action of a force.

Quantitatively - the pieces of tape are distant from the top 1, 4, 9 and 16 units. These are 1^2 , 2^2 , 3^2 and 4^2 , so if the clicks are evenly spaced in time, the distance d travelled is proportional to the square of the time $d = kt^2$. The steeper the slope, the larger the acceleration, but the clicks are still even, so $d = k't'^2$, with a new constant k' .

Experiment 1.22 - Acceleration due to g .

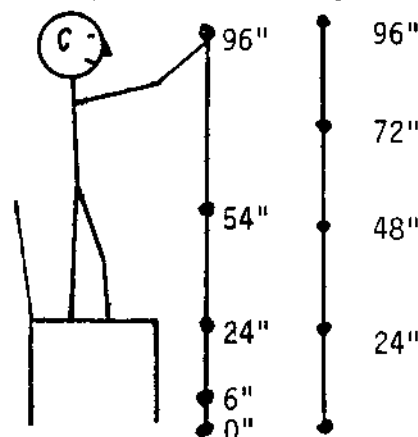
Materials: - string, marbles, tape, ruler

Method: - The previous experiment, using marbles on a ruler, is not convincing to some students. The following experiment is a little simpler, and employs g directly.

Two strings are cut the height of the room, or as high as the student can reach standing on a chair. We will suppose this is eight feet. Marbles are attached to one string at two foot intervals. The other string has marbles attached at intervals proportional to the squares of the whole numbers, as follows

number	square	multiplied by 6 difference	
0	0	0	
1	1	6"	6"
2	4	24"	18"
3	9	54"	30"
4	16	96"	42"

Now stand on a chair holding the two strings as shown in the following figure



The bottom marble should just touch the floor. Drop the string having marbles at a uniform spacing. Now drop the other string. The clicks are easier to hear if you drop the string into a trash can.

Qualitative Question: - Do you hear the time between clicks get shorter as the higher marbles from the uniformly spaced string strike the floor?

Does the time between clicks for the non-uniformly spaced string stay the same?

This is because the higher marbles are accelerated for a longer time and are traveling faster, covering the same distance in a shorter time as they approach the floor than the marbles starting near the floor.

Quantitative Question: - The formula for acceleration under g is

$$\text{distance travelled } S = \frac{1}{2} g t^2 \quad (t = \text{time taken})$$

By spacing the marbles at intervals such that the square root of successive distances is proportional to whole numbers, the time between successive clicks must be constant. In our case this time is, taking the lowest marble, $\frac{1}{2}$ foot.

$$\frac{1}{2} = \frac{1}{2} \times 32 t^2$$

$$\text{therefore } t = \frac{1}{\sqrt{32}} = .176 \text{ seconds} = \text{time between clicks}$$

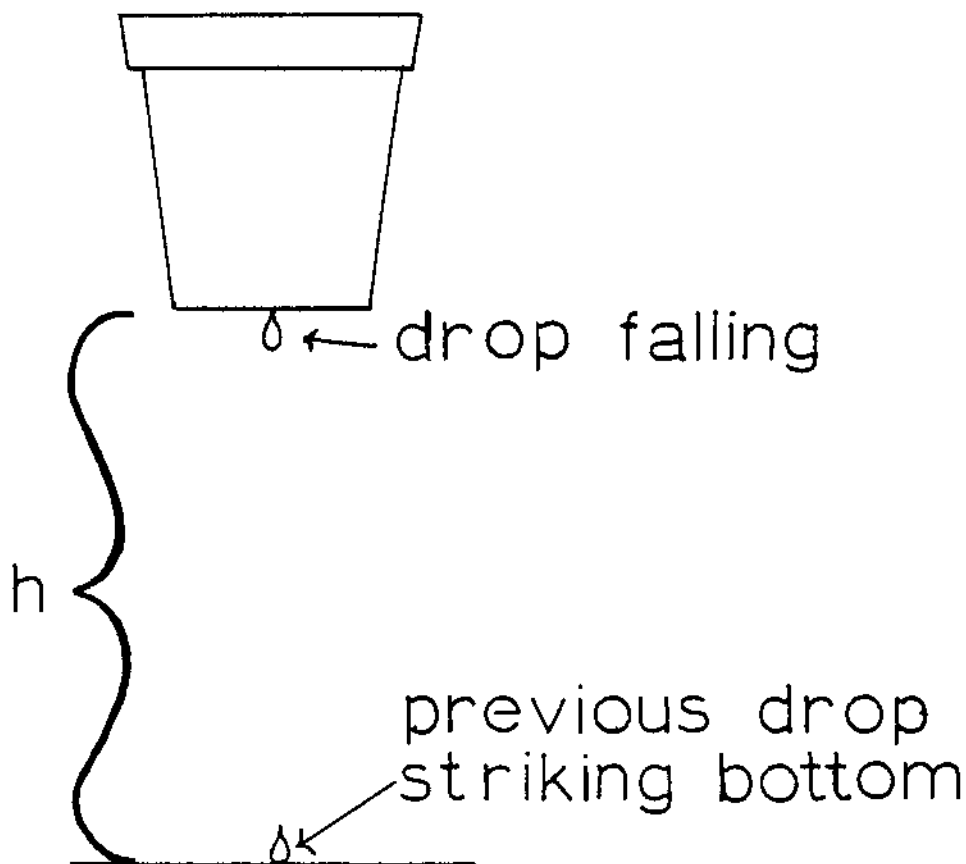
Shift one of the marbles up or down the string, to tell how sensitive your ear is to changes in timing between clicks. A change in position of 20% in any one marble gives a quite noticeable non-uniformity between clicks.

Another way of measuring g is to poke a very small hole in the bottom of a cup. The drops fall onto a trash can, a sheet of paper, or anything that makes a noise. Move the cup up and down until the sound of the previous drop striking the can exactly corresponds to the time a drop leaves the cup. Then measure the height above the bottom of trash can, or sheet of paper, and the time for say 50 or 100 drops to fall. You can get the time it takes the drop to fall the fixed distance from this.

Time for 100 drops, T , = sec.

Height of cup, h , = m.

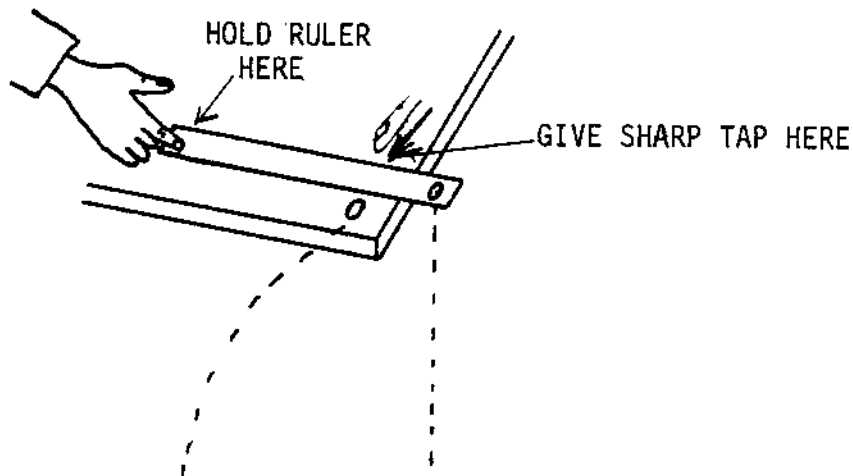
$$g = \frac{2.0000h}{T^2} = \quad \text{m./sec}^2$$



Experiment 1.23 Independence of vertical and horizontal motion

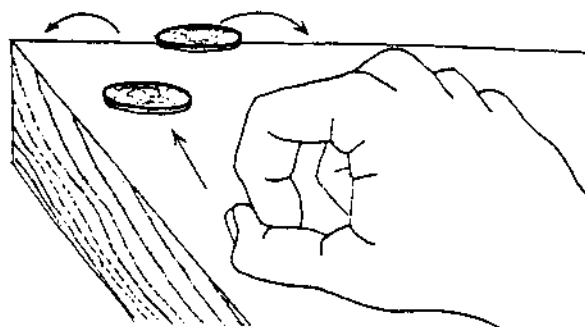
Materials required - Foot ruler, two coins or two pieces of chalk.

Procedure: - at a corner of the table, place a piece of chalk, or better, a coin. Hold a ruler behind it, as shown, and a piece of chalk, or another



coin on the end of the ruler, over the edge of the table. Give the ruler a sharp tap. The coin on the ruler, left behind by its inertia, falls vertically. The coin on the table has a large horizontal velocity (but no vertical velocity). Both hit the ground simultaneously, as one can easily hear. You can do this experiment without the ruler by getting one coin to nudge the other off the table as shown.

What do we learn?



This demonstrates horizontal and vertical motion are independent of one another. Both coins start with no vertical velocity, even though one has a large horizontal component, both accelerate at the same rate and reach the ground together.

Experiment 1.24. Two dimensional kinematics using coins

Materials required: Two coins - two nickels (or quarters or pennies), and a nickel, a quarter and a penny. A piece of paper and pencil, or some other way of marking the coin's position on the floor.

Procedure: Collisions in two dimensions can be studied using coins on a sheet of paper. First, let us look at the angles in the collision process. Put one coin a few inches behind a similar coin (pennies or nickels are good) on a sheet of paper, and draw round them, marked 1 and 2 in figure 1. Flick one coin at the other, so that they travel a fair distance but do not leave the paper. You can propel the coin either by using a ruler rather like a pool cue, or by holding it at one end so that it lies perpendicular to the direction in which the coins should travel, then giving the other end, close behind the coin, a sharp tap. Draw around the coins in their final position. This is marked 3 and 4 in figure 1, which was an actual experiment using nickels. The rotational motion occurring during the collision may be studied by noting the orientation of the two coins before they start (e.g. the position of the "L" in "Liberty" on the head side of the coin) and when they finish. Draw lines 5 and 6, extending them backwards as shown, so that these lines are tangent to the original and final position of the struck coin. Place the initial coin between these lines so that it touches 2, and draw around it, as the dotted circle 7. This is the position of the striking coin when the collision occurred. Now draw lines 8, 9, 10 and 11, extending 8 and 9 so that the angles θ and ϕ can be recorded by a protractor. The distances d_1 traveled after collision by the striking coin, and d_2 , traveled by the struck coin should also be measured. V is taken as the velocity of the first coin just before the collision, and v_1 just after the collision, with v_2 the velocity of the second coin just after the collision. It is convenient to use a set of axes parallel (y) and perpendicular (x) to the direction of collision, which is that of v_2 . This selection of axes has the advantage that to a first approximation no angular momentum is transferred between the coins from the effects of the collision in the y direction, and only angular momentum is transferred in the collision by motion in the x direction. Furthermore, since there is no linear transfer of momentum in the x direction, our consideration of any inelasticity in the collision can be restricted to the y axis, which is parallel to the line of impact. Conservation of momentum in the x direction then gives us

$$V \sin \theta = v_1 \sin (\theta + \phi) \quad (1)$$

and in the y direction

$$V \cos \theta = v_2 + v_1 \cos (\theta + \phi)$$

At this point we must consider the energy equation. The audible click as the coins collide shows an energy loss on collision, a measure of which is the coefficient of restitution e , defined as the ratio of the difference in velocity of the colliding objects along the line of impact (i.e. the y direction in our case) after the collision to that before the collision. Experimentally, this is found to be a constant. Considering now the motion in the y direction

$$\begin{aligned} e &= (v_2 - v_1 \cos (\theta - \phi)) / V \cos \theta \\ &= (V \cos \theta - 2 v_1 \cos (\theta - \phi)) / V \cos \theta \end{aligned}$$

$$\begin{aligned}
 & (\text{Since } v_2 = V \cos \theta - v_1 \cos (\theta + \phi)) \\
 & = 1 - 2v_1 \cos (\theta + \phi) / V \cos \theta \\
 & = 1 - 2 (V \sin \theta / \sin (\theta + \phi)) \cos (\theta + \phi) / V \cos \theta \\
 & (\text{Since } v_1 = V \sin \theta / \sin (\theta + \phi)) \\
 & = 1 - 2 \tan \theta / \tan (\theta + \phi)
 \end{aligned}$$

If $e = 1$, the collision is perfectly elastic, $\tan \theta / \tan (\theta + \phi) = 0$, so $\theta + \phi = 90^\circ$. If $e = 0$, the collision is perfectly inelastic, $2 \tan \theta = \tan (\theta + \phi)$ and $\theta = \phi = 0$.

For the example shown in figure 1, $\theta = 31^\circ$, $\phi = 35^\circ$, $e = .46$. We have found collisions between nickels to be rather more reproducible than with other coins.

Having obtained the coefficient of restitution from the angles, we can now examine the energy equation. If the coefficient of sliding friction is constant, independent of the velocity, the distance travelled by the coins should be proportional to their energy after the collision. If the distances are d_1 and d_2 as shown in figure 1, m is the mass of one coin and μ the coefficient of friction.

$$mg\mu d_1 = \frac{1}{2} m v_1^2$$

$$mg\mu d_2 = \frac{1}{2} m v_2^2$$

Since the relative velocity of the two coins in the y direction before the collision is

$$V \cos \theta = v_2 + v_1 \cos (\theta + \phi)$$

and after the collision is $v_2 - v_1 \cos (\theta + \phi)$

$$e(v_2 + v_1 \cos (\theta + \phi)) = v_2 - v_1 \cos (\theta + \phi)$$

$$\text{or } v_2 (1 - e) = v_1 (e \cos (\theta + \phi) + \cos (\theta + \phi))$$

$$v_1/v_2 = (1 - e) / (1 + e) \cos (\theta + \phi)$$

We can check this against the ratio $\sqrt{d_1/d_2}$. The results are generally not very reproducible because the frictional forces are not constant. In the case shown $\sqrt{d_1/d_2} = \sqrt{10 \text{ cm}/10.6 \text{ cm}} = .97$.

The angular momentum exchanged in the collision may justifiably be ignored, provided the collision is not too far from head on. As the collision becomes more glancing, the effects of angular momentum become more pronounced. If the coins are sufficiently smooth, they will slide on colliding, and again no angular momentum will be exchanged. However, if they stick, a tangential force

F will tend to rotate both coins in the same direction. This force may be replaced by a couple C plus a force F acting through the center of mass of the coin, such that $C = Fr$, where r is the radius of the coin, as shown in Fig. 4. The torque impulse leads to an angular velocity ω in the same sense for the two coins:

$$I\omega = Ct = Frt$$

where I, the moment of inertia of the coin is $\frac{1}{2}mr^2$ and t is the time for the collision. Since F is in the x direction, the change in momentum

$$mv_x = Ft, \text{ so } I\omega = mv_x r \text{ or } \omega r = v_x \quad (1)$$

where v_x is the change in velocity in the x direction during the collision.

To see how large this effect is in practice, we can employ the results of the experiment, shown in fig. 1. Let the angle of rotation of the struck coin be θ_2 before it stops, and of the incident coin θ_1 . The frictional forces between the paper and the coins are independent of velocity, so there will be no net torque on the coins after the collision. Using the previous nomenclature.

$$d_1 = \frac{1}{2}at_1^2 = \frac{1}{2}\mu g t_1^2$$

$$d_2 = \frac{1}{2}\mu g t_2^2$$

where a is the deceleration and t_1, t_2 the time for the respective coins to stop after the collision

$$\theta_2 = \omega_2 t_2 = \omega_2 \sqrt{2d_2/\mu g}$$

$$\omega_2 = \theta_2 \sqrt{\mu g/2d_2}$$

Using the values from fig. 2, $\theta_2 = 130^\circ$, $d_2 = 10.1$ cm and $\mu = .24$, we find $\omega_2 = .775$ rads/sec. From equation 1, $\omega_2 r = 2v_{2x} = .004$ m/sec.

This represents only five percent of v_{2y} and can be neglected to the degree of accuracy to which we are working.

It is possible to carry out the experiments using the transparent film on an overhead projector, drawing around the coins with a suitable marking pen. However, the coefficient of friction with the film is higher (about 0.4) and not as constant. The velocities at the time of collision may be estimated if the coefficient of sliding friction between the coin and the paper is known. This may easily be measured by tilting a pad of paper and dropping the coin at the top. If the coin slides barely all the way down without stopping, the tangent of the angle η the pad makes with the table is the value of the coefficient of friction. From figure 2, the reaction of the pad on the coin is $mg \cos \eta$, and the force parallel to the pad is $mg \sin \eta$, so $\mu = mg \sin \eta / mg \cos \eta = \tan \eta$. μ is generally about .24 between coins and paper, so traveling a distance of 10 cm after collision would require an initial velocity of

$$\frac{1}{2} mv^2 = mgud = m \times 9.81 \times .24 \times .1$$

$$\text{so } v = .7 \text{ m/sec}$$

or about 1.5 mph.

The angle can be measured using a ruler or a protractor, as shown in figure 2.

More accurate dynamics experiments can be obtained by placing the second coin right at the edge of the table, so that both it and the incident coin are knocked off the table onto the floor, as shown (fig. 3). A point directly beneath the coin at the table edge is marked on the floor, by dropping a coin there and seeing where it lands. A second coin is placed a few inches behind the first, and is propelled or flicked toward it in a direction normal to the edge of the table, as described earlier. The coins collide at some angle. Now, mark on the floor with pen, pencil or chalk where both struck the floor.

Make a diagram, as before, for the distances and angle. It is easier if two people mark on the floor where they see the coins fall, using chalk or pencil, and for permanence, use a large sheet of paper, such as a newspaper. Do this with two nickels, a nickel and a quarter, a nickel and a dime, etc. The masses of the various coins is given in experiment 1.01 p7

Qualitatively we learn:

1) When like coins collide, both fall off the table (except for a head-on collision) showing both travel forward.

If the collision were perfectly elastic, the angle between the trajectory of the coins would be 90° , and a head-on collision would leave one coin poised on the edge of the table. The mere fact that we hear a click as the coins collide shows that energy is being lost, and the coins travel off with an angle less than 90° between them.

2) When a heavy coin strikes a light one, both leave the table at considerable speed, even for a head-on collision.

3) When a light coin strikes a heavy one, the light coin stays on the table unless it delivers the heavy coin a glancing blow. Unlike (2) where the forward momentum is shared with the struck coin, here the struck coin carries the forward momentum with it, and the incident coin may recoil. Let us try to get a simple intuitive picture of this process. Imagine you are

sitting at the center of mass of the system, observing the two coins approaching you, the ratio of the speeds being inversely proportional to the masses, so the heavy coin approaches slowly, and the light coin rapidly. They collide where you are sitting, reverse, and go off with the same speed as before, apart from what is lost in the collision. The center of mass, where you are sitting, moves steadily forward during the whole of this process. Combining this center of mass motion with that of the coins about the center of mass, we see that after the collision, if the light coin is first, it has a fast forward motion superimposed on that of the center of mass, and will move rapidly forward, whereas the heavy coin moves forward sluggishly. If the heavy coin is first, the light coin will move backward, since its reverse speed is so much larger than that of the center of mass.

We can use our measurements to obtain a quantitative check on the theory. For example, if the table is h cm high (an average is about 74 cm) the coin takes a time $t = \sqrt{2h/g}$ to reach the floor, which is 0.39 seconds. If the horizontal velocity of the coin is v , it will travel a distance $0.39v$ horizontally before striking the floor. Since the horizontal component of momentum of the system parallel to the edge of the table is zero, the two coins, of masses m_1 and m_2 have equal and opposite momenta parallel to the edge. Hence

$m_1 u_1 + m_2 u_2 = 0$, where u_1 and u_2 are the respective velocities parallel to the edge of the table as shown in figure 4. Since $u_1 t = \ell_1$ and $u_2 t = \ell_2$, where ℓ_1 and ℓ_2 are the distances the coins travel in a direction parallel to the table edge, $m_1/m_2 = \ell_1/\ell_2$. Check whether this is true--it is independent of the elasticity of the collision since it depends on conservation of momentum. For an elastic collision between like coins, a rectangle can be drawn as shown, and $\ell_1 = \ell_2$. (Fig.3)

The coefficient of restitution e and the ratio of v_1/v_2 may be obtained from the angles, as for the coins sliding on the table. The values of v_1 and v_2 obtained directly from the impact points of the coins on the floor are more accurate than those obtained from sliding distances, because frictional forces vary somewhat.

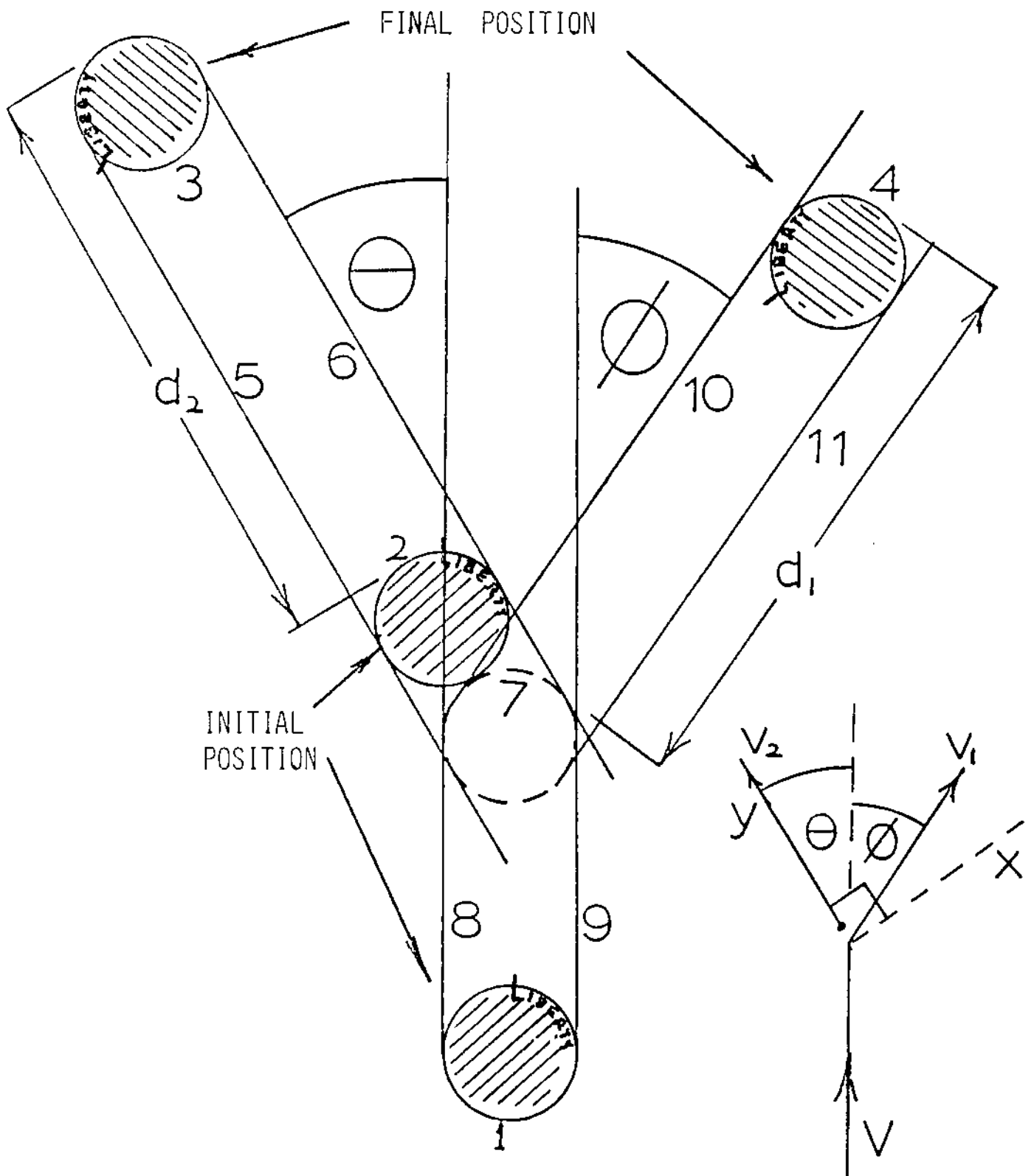


FIG. 1

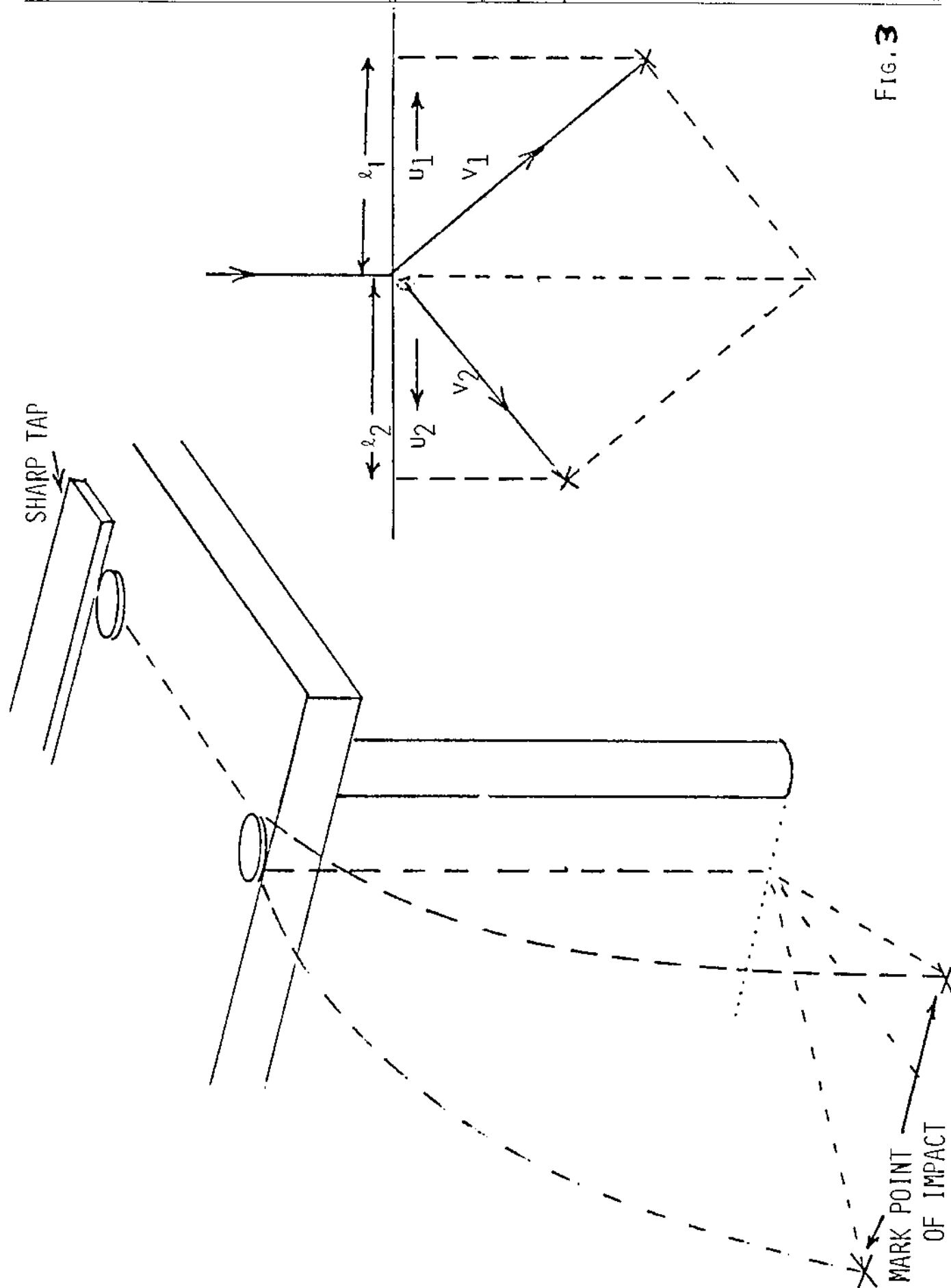


FIG. 3

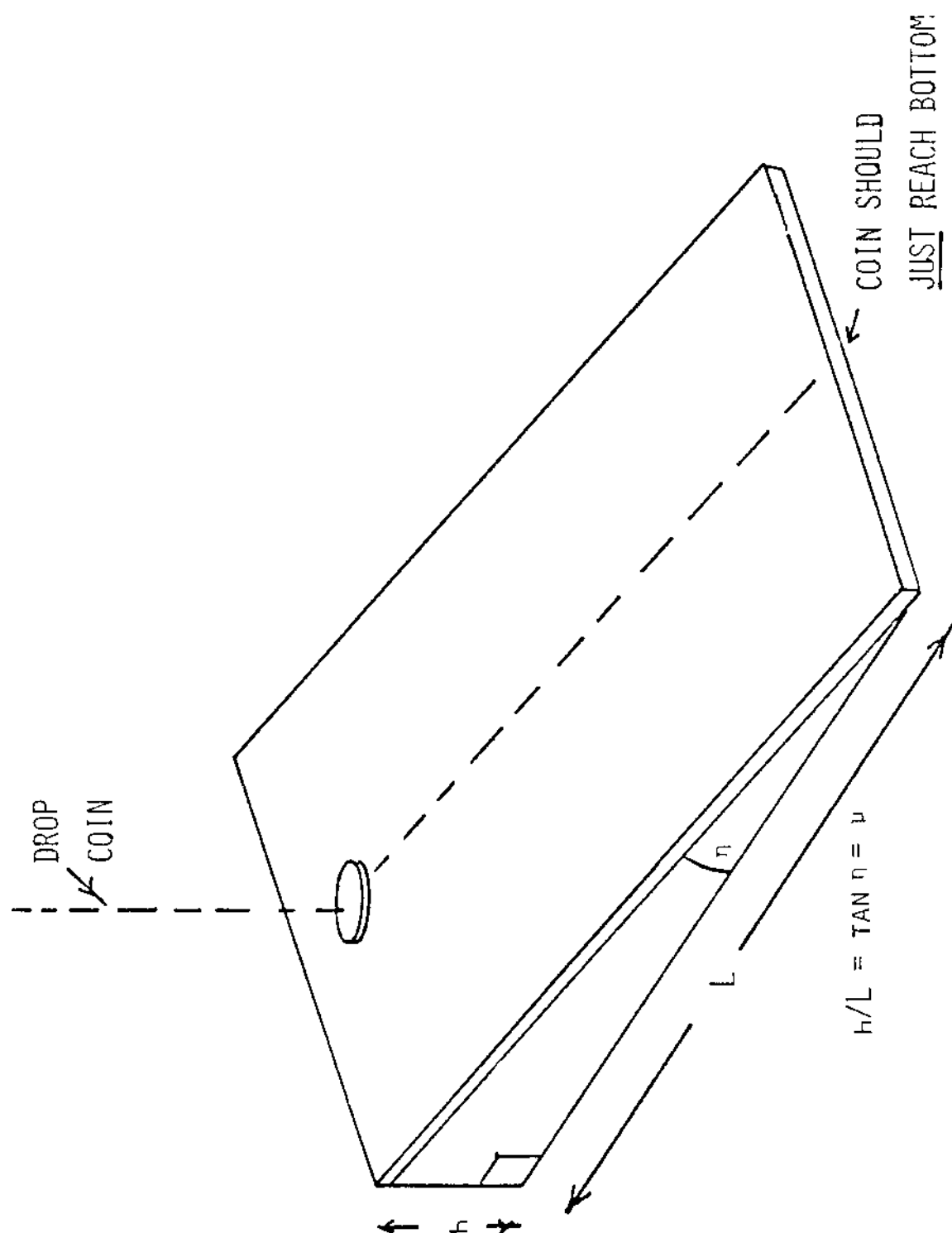


FIG. 2.

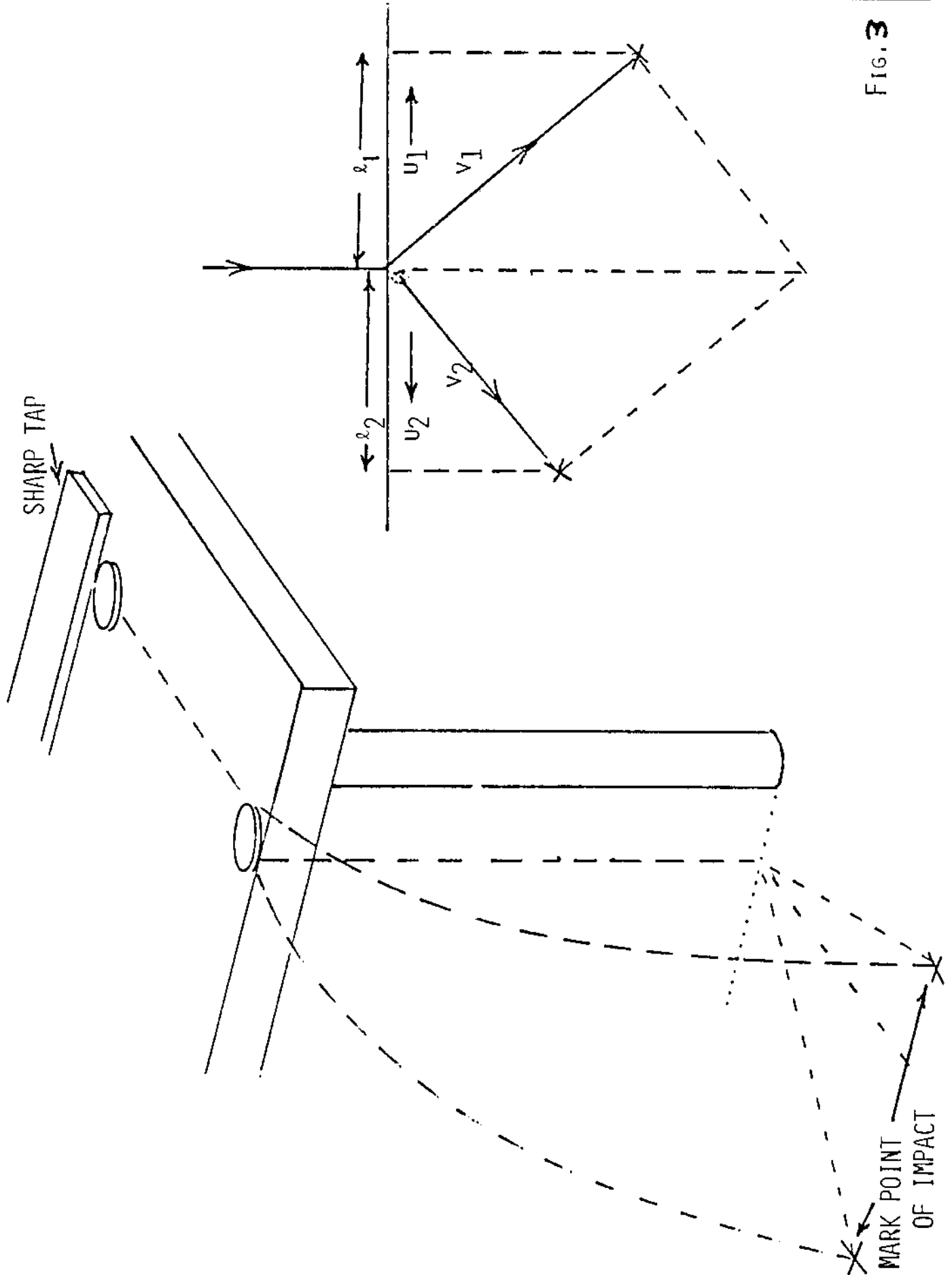


FIG. 3

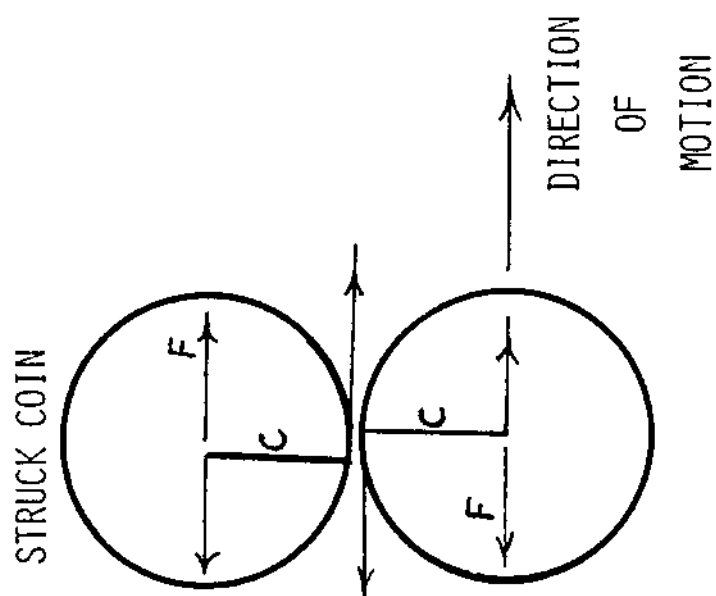
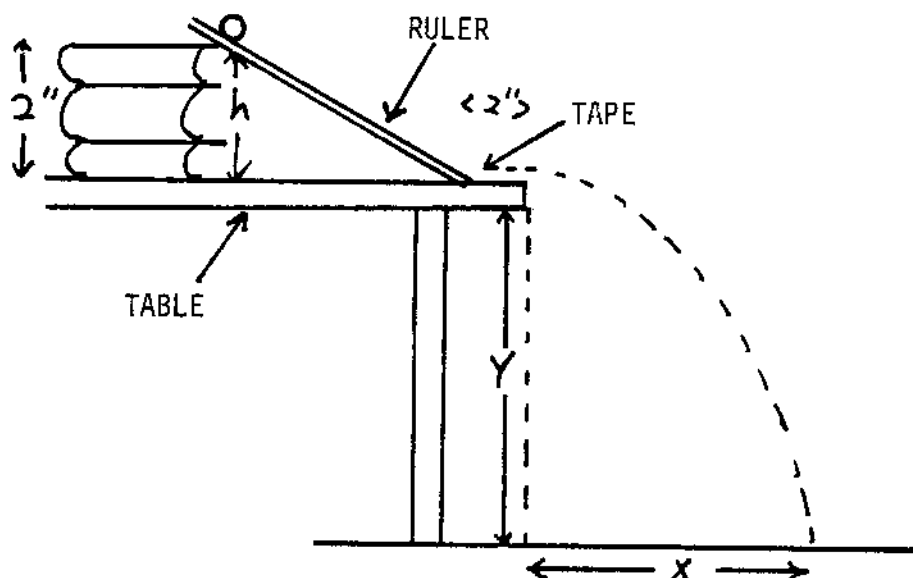


FIG 4

Experiment 1.25 Kinetic and Potential Energy

Materials: Ruler, two marbles, paper.

Procedure: We wish to study the relationship between potential and kinetic energy. Prop the ruler up against a book, as shown, so that the end is about two inches from the edge of the table, and the other end is about two inches above the table. Tape the lower end of the ruler to the table, to stop it from slipping.



Drop a marble from the edge of the table, and make a mark on a sheet of paper taped to the floor, where it falls. Now, release the marble from the 3", 6", 9" and 12" marks on the ruler, and again note where they strike the floor, marking a sheet of paper taped to the floor. Repeat each measurement a few times, to check. You will need to know:

1. The height of the marble off the table.
2. The height of the table off the floor.
3. The distance the marble travels horizontally from the edge of the table to the point where it strikes the floor. i.e. the distance marked x on the diagram.

Qualitative: The marble has potential energy as it is held at the top. This potential energy is converted into kinetic energy as the marble rolls down the ruler and along the table. The marble leaves the table horizontally, so it has no vertical motion. Since we can separate vertical and horizontal motion, it always takes the same time to reach the floor from the table.

The potential energy increases as we move the marble up the ruler. This gives a larger kinetic energy, and the additional horizontal velocity resulting makes the marble travel farther horizontally before striking the floor.

Quantitative: The potential energy of the marble above the table is mgh . The kinetic energy of the marble as it leaves the table is equal to this

$$mgh = \frac{1}{2} m v^2$$

so the horizontal velocity is $v = \sqrt{2gh}$
 where g is the acceleration due to gravity = 981 cm/sec^2 or 32 ft/sec^2 .

The time taken to strike the floor from the table, since initially there is no vertical motion, is given by

$$y = \frac{1}{2} gt^2$$

now $x = v_x t$

so $v_x = \sqrt{2gh}$, $t = \sqrt{\frac{2y}{g}}$

$$x = 2 \sqrt{hy}$$

$$x^2 = 4 hy$$

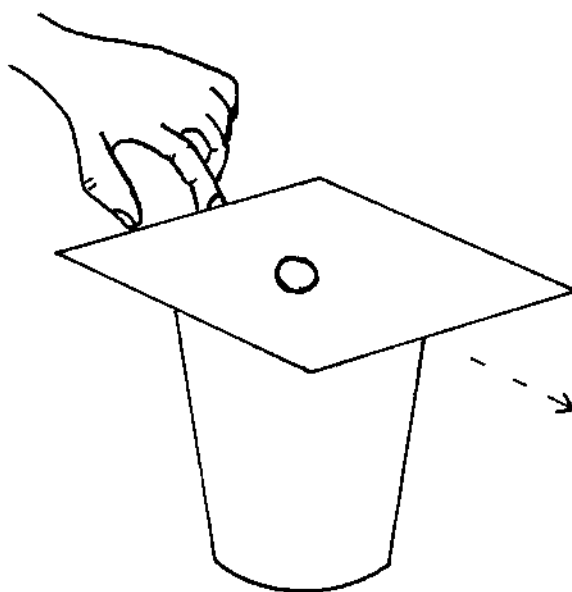
Calculate x from h and y . Do the calculated and experimental values agree? Is the measured value for x larger or smaller than that calculated? Why?

Experiment 1.26 Inertia (Newton's 1st Law)

Materials needed - Dixie cup, card (or paper), piece of chalk or a coin.

Procedure: Place the coin or chalk on the card on the top of the cup, as shown. With the forefinger, tap the card sharply. It will slide off the top, and the coin will fall into the cup. This is a less drastic version of the table-cloth trick, where the cloth is pulled from under a set table (always pull the cloth downward).

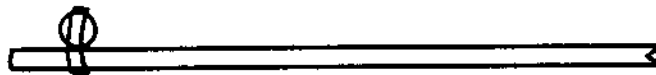
Qualitative: "A body continues in its state of rest or uniform motion, unless acted upon by a force". In this case, negligible force acting on the coin when the card is removed, so its inertia keeps it where it is.



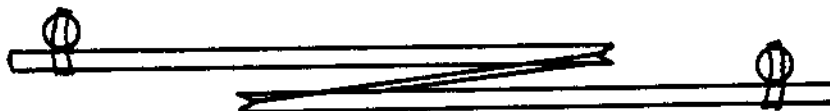
Experiment 1.27 Action and Reaction

Materials: Straws, marbles, rubber band, pencil, scissors, sticky tape

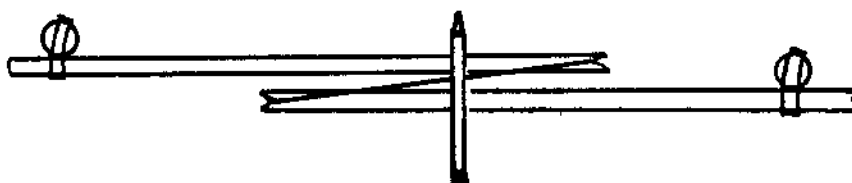
Procedure: Take two marbles, and attach each one to the end of a straw, using sticky tape. Cut a very small notch at the other end of the straw. Now place a



rubber band in the notch, stretch it, and place the notch of the other straw in the other end of the loop of the band, so it appears as shown.



Place the system on a smooth table, or a smooth floor, and hold it in the loaded condition using a pencil. Suddenly release the pencil--the marbles will fly



apart and travel across the floor. Measure the distance they travel. Do this for several tensions in the rubber band. Now attach a second marble to the end of one straw, and repeat the experiment.

Do not try to hold the straws down with the fingers--the fingers are too sticky and flexible to release straws at exactly the same time. It is possible to hold the straws in the tense position with a strip of cellotape, which is gradually pulled off by the rubber band until the system springs apart on its own. However, some effort is required to find the right piece of tape which is not too small that the band releases too early, nor too big so it never releases at all.

You can make a mark on the straw if you want to repeat the experiment with the same tension; draw the band to the same mark.

Qualitative

Questions: With just one marble on each straw, is the distance each travels the same? Why?

With two marbles, is the distance the same? Why?

Quantitative: The distance traveled is not exactly equal each time. Why? Is the ratio of distances more nearly one under high or low tension? Why?

With the two marbles on one straw, and one on the other, is one distance twice the other? Why? Is the ratio of the distance constant, independent of rubber band tension?

Experiment 1.28 Weightlessness

Materials: rubber bands, two coins, paper clip, tape, water

Method: It is difficult for students to understand the apparent absence of force in free fall. Here are two simple experiments

Two little weights are hung by rubber bands over the edge of a styro-foam cup. The weights used were small chemical balance weights, but instead, coins such as quarters or half dollars can be taped to rubber bands, leaving about an inch of free tape as shown in Fig. 1. The other ends of two rubber bands go through a hole in the bottom of the cup, and are held there by a paper clip. The bands should be under light tension. Now drop this system, and the weights hop into the cup. This is because the coins become "weightless" when dropped while the tension remains in the rubber bands. In the second experiment, which is even simpler, you poke two holes close to the bottom of a paper cup, as shown. The cup is filled with water, which pours out of the two holes. Now climb a step ladder, hold the cup high at arms length, and drop it, preferably into a trash can. No water runs out as the cup falls-- again, the water is "weightless" in the falling system. Gravitational forces are absent in the accelerated system.

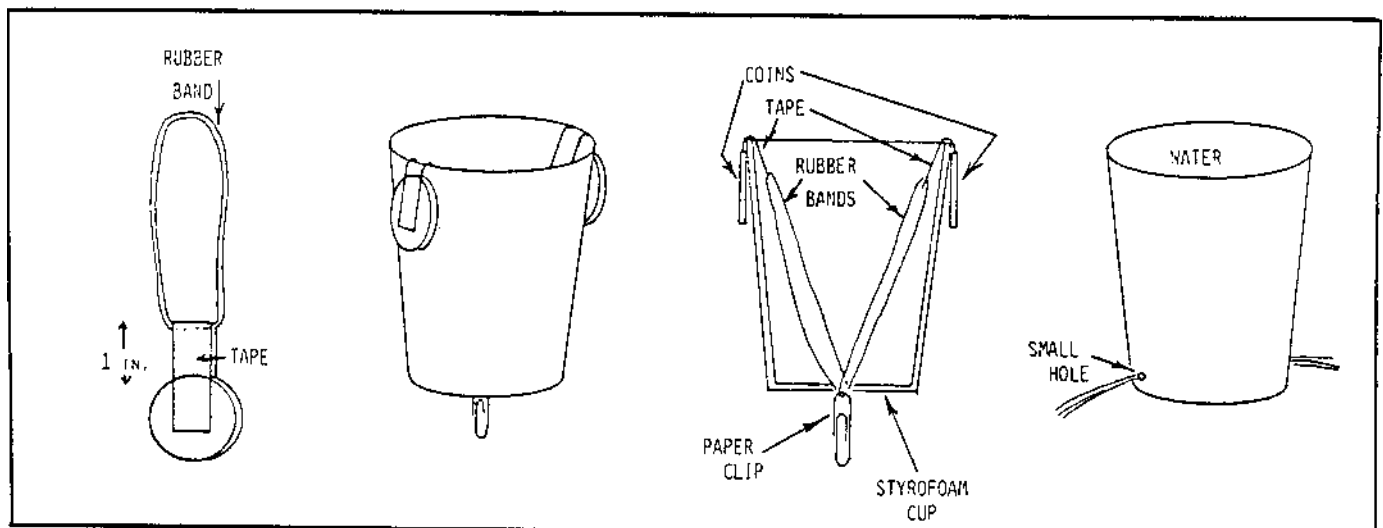
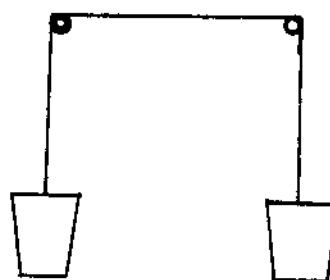
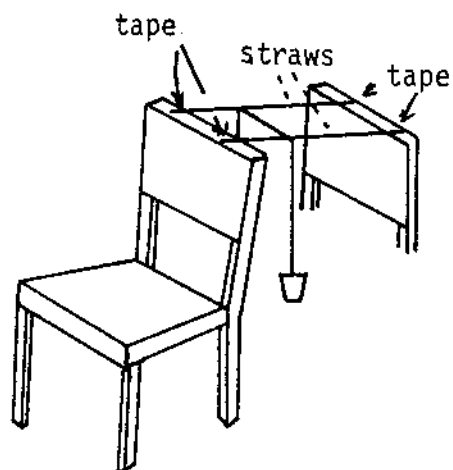


Fig. 1.

Experiment 1.29 Atwood's machine

Materials required: Two drinking straws, two foam or paper cups, marbles, string.

Procedure: Attach the cups to the ends of the string, as shown, and fasten the straws with tape to the backs of two chairs. Now, put 6 marbles



_____ ground

in one cup, and four in the other, and release the cups. Notice the heavier one moves faster and faster till it strikes the floor. Add marbles to the heavier cup, and again release.

Qualitative: The greater the difference in number of marbles between the two cups, the greater the acceleration - if the number of marbles is equal, there is no acceleration.

Quantitative: The mass being accelerated is the sum of the masses in the two cups, but the force accelerating them depends on the difference between the masses. Hence the acceleration is less than g , the acceleration due to gravity, by $g \frac{(n_1 - n_2)}{(n_1 + n_2)}$ where n_1 and n_2 are the number of marbles in the two cups. If you have a watch, time how long it takes the cup to reach

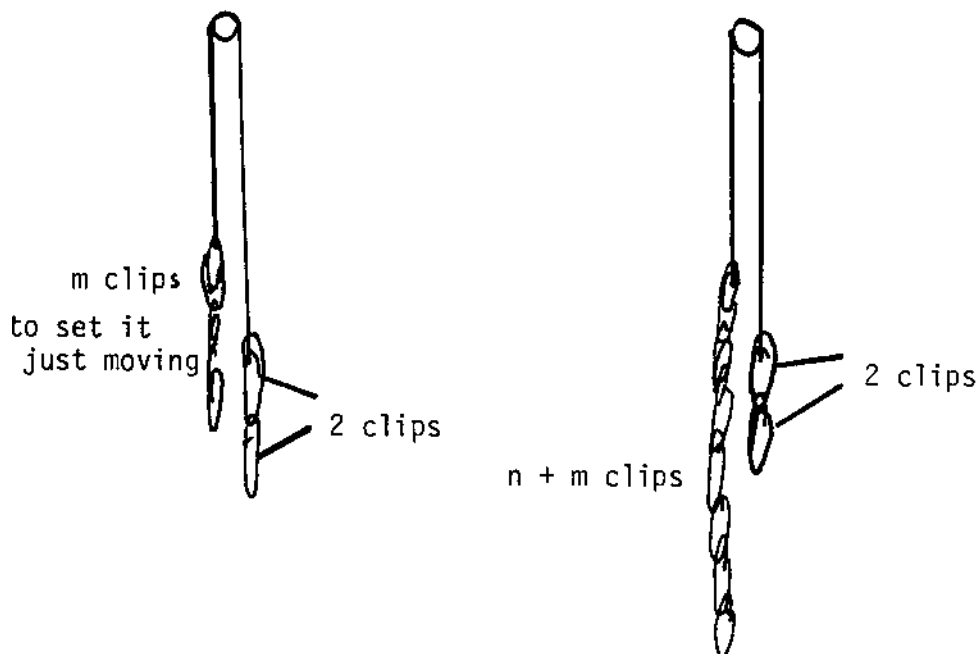
the floor, with different numbers of marbles in the two cups. Measure the distance to the floor. Calculate the acceleration from the formula.

$$\text{distance to floor} = \frac{1}{2} \text{ acceleration} \times (\text{time})^2$$

Use the previous formula to calculate g .

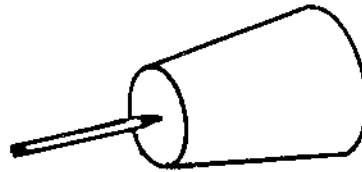
Note: There is considerable friction between the string and the straws. To reduce this, it is recommended that the pulley described on the next page be employed in place of the straws.

An even simpler version of this experiment uses paper clips. Hang two paper clips by a thread over a straw as shown, then add clips to the other side until, when set going, they just continue to move. Now add a few more clips to this side, and measure the time to reach the floor. The accelerating force is due to the added paper clips, or mg , and the mass accelerated is the total number of clips.

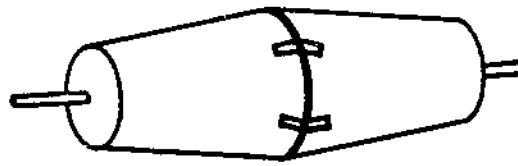


Construction of an (approximately) massless, friction free pulley.

Take two styrofoam cups, and poke a pencil point through the exact centers of the bottoms

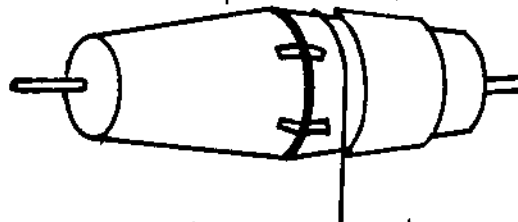


Now, push a straw through the bottom of one from the outside, and the bottom of the other through the inside, so the the two open ends face.

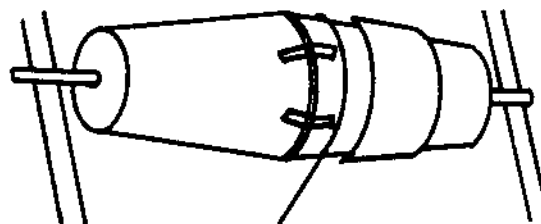


Tape the two faces tightly together with several pieces of sticky tape

Now, cut the bottom off another cup and slide it over one of the two.



The pulley string runs between the two cups shown, and the straw is supported between the backs of two chairs, two tables, two stools, two desks, or anything convenient. The straw rolls, so that friction is very low, and the mass of the two and a half cups is very small indeed.

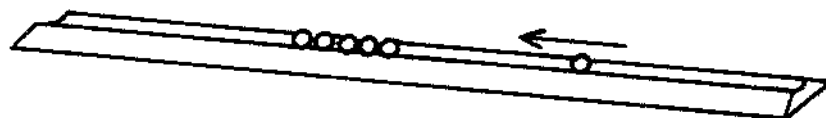


Using this for Atwoods machine, the cups carrying marbles must have their tops cut off, or they will bump.

Experiment 1.30 Collision of spheres -

Materials: marbles, string, tape, ruler

Procedure: Place five marbles in the groove at the center of the ruler and flick a sixth rapidly at the bunch.



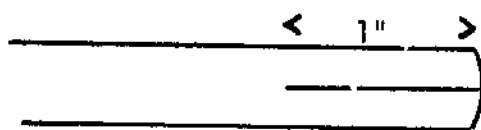
Qualitative question: What happens? Why does only one marble move?

If you roll the odd marble slowly down the groove, it may not stop dead on hitting the others, especially if there are only one or two and not five. Why? It is because the linear, but not the rolling motion is transferred.

This simple experiment cannot be used to find collision energy loss, since too much energy is dissipated in friction to the groove.

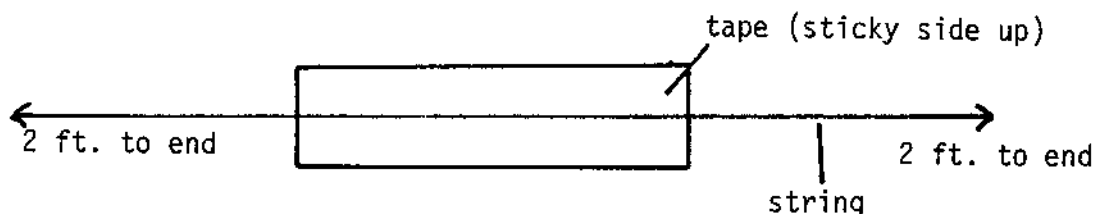
To study collisions between two marbles, a ballistic pendulum is used.

Take a straw, and cut slits at each end one inch deep

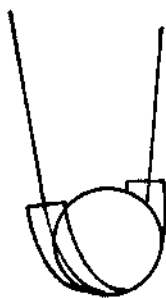


Cut two lengths of string four feet long, cut two pieces of tape 1 1/2" long.

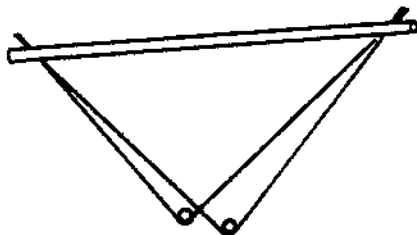
Stick the center of the string along the center of the tape.



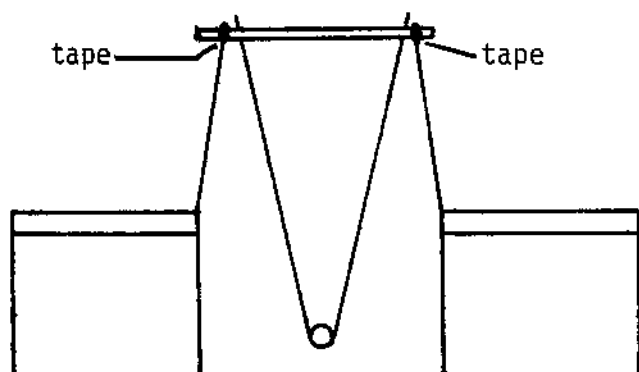
Now place a marble at the center of the tape, and press the tape to hold the marble like a stirrup



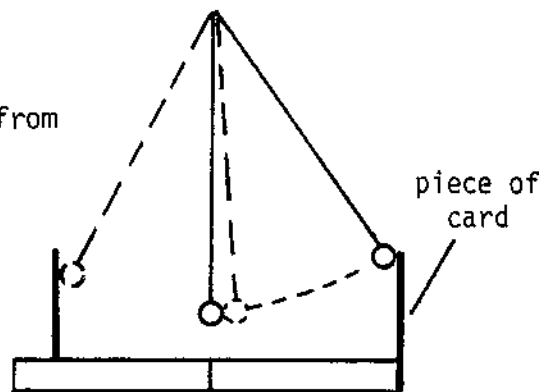
put the ends of the string in the ends of the straw, so there are two pendulums as shown below



Now comes the tricky part. Carefully pull the strings through the slits until they are of approximately the same length. It is important that, when the two marbles collide, they must hit one another as close as possible to dead center. Attach the ends of the straws, using tape, to the backs of two chairs, or tables, at the same level. Hold the ruler so that



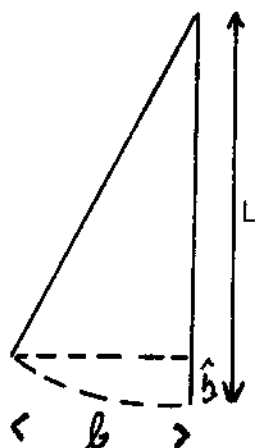
observe from
the side



the 6" mark lies between the two marbles. Pull one back to 0 and release. How far does the other one travel? Make several experiments, and measure this as accurately as possible.

Qualitative: Why does the struck marble not travel as far, or rise as high as the incident marble? Where does the energy go? Are there frictional losses?

Quantitative:



The incident marble drops a distance h before colliding. From geometry

$$(2L-h)h = \ell^2 \text{ or } 2Lh = \ell^2$$

Now, the kinetic energy just before collision $= \frac{1}{2} mv^2 = mgh$, the original potential

$$\text{energy } \frac{1}{2} mv^2 = mg \frac{\ell^2}{2L} \quad \text{or} \quad v = \ell \sqrt{\frac{g}{L}}$$

After collision, the ball travels a distance ℓ' where $v' = \ell' \sqrt{\frac{g}{L}}$

$$\text{so } \frac{\text{velocity before collision}}{\text{velocity after collision}} = \frac{\ell \text{ before collision}}{\ell' \text{ second marble after collision}}$$

if the first ball is reasonably stationary after collision. The ratio of the relative velocity of the spheres before and after impact is called the coefficient of restitution, R and measures how inelastic the collision is.

What would it be for a perfectly elastic, and perfectly inelastic collision?

How much energy is lost in the collision?

Note that if the string is 61cm (2ft) long, and the marble's mass is 5 gms., the initial stored energy on drawing it back 15.24 cm. (i.e. 6") is

$$5 \times 981 \times \frac{(15.24)^2}{2 \times 61} = 9,344 \text{ dynes } (0.09344 \text{ N})$$

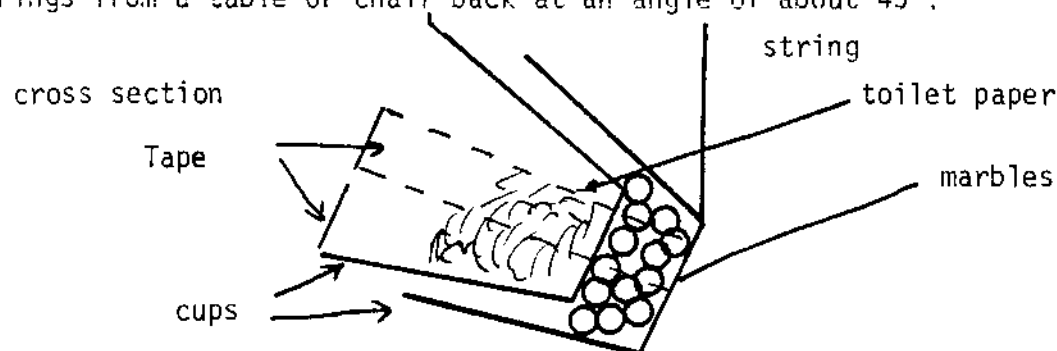
The energy lost is then $9,344 (1 - R)^2$ ergs. $(9.344 \times 10^{-4} (1 - R)^2 \text{ J})$

Experiment 1.31 The Ballistic Pendulum

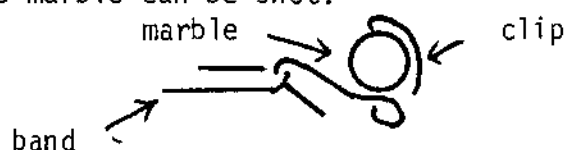
Materials: tape, marbles, string, rubber band, paper clip, cups

The ballistic pendulum is used to measure the speed of high speed projectiles by shooting the light projectile into a heavy hanging mass.

Procedure: Put about twenty five marbles in a cup, place another cup on top, and tape the two together, wrapping the tape all around, leaving a space at the top a little less than half to shoot in a marble. Tuck some toilet paper in the outer cup to stop the incoming marble. Hang this setup by two parallel strings from a table or chair back at an angle of about 45°.



Now, bend a paper clip (as below) to hold a marble, and attach a rubber band so the marble can be shot.



The other end of the band can be held by another clip, and the marble catapulted into the cup. It is easiest to shoot off a pile of books, as shown. The distance the cups travel is measured by a ruler underneath. You can fold a paper indicator to be pushed by the cups.

Measure the speed for different, measured stretches of the rubber band.

The analysis is given in experiment 1.27.

If L is the length of the string, ℓ the distance the cup travels, the initial velocity of the cup is

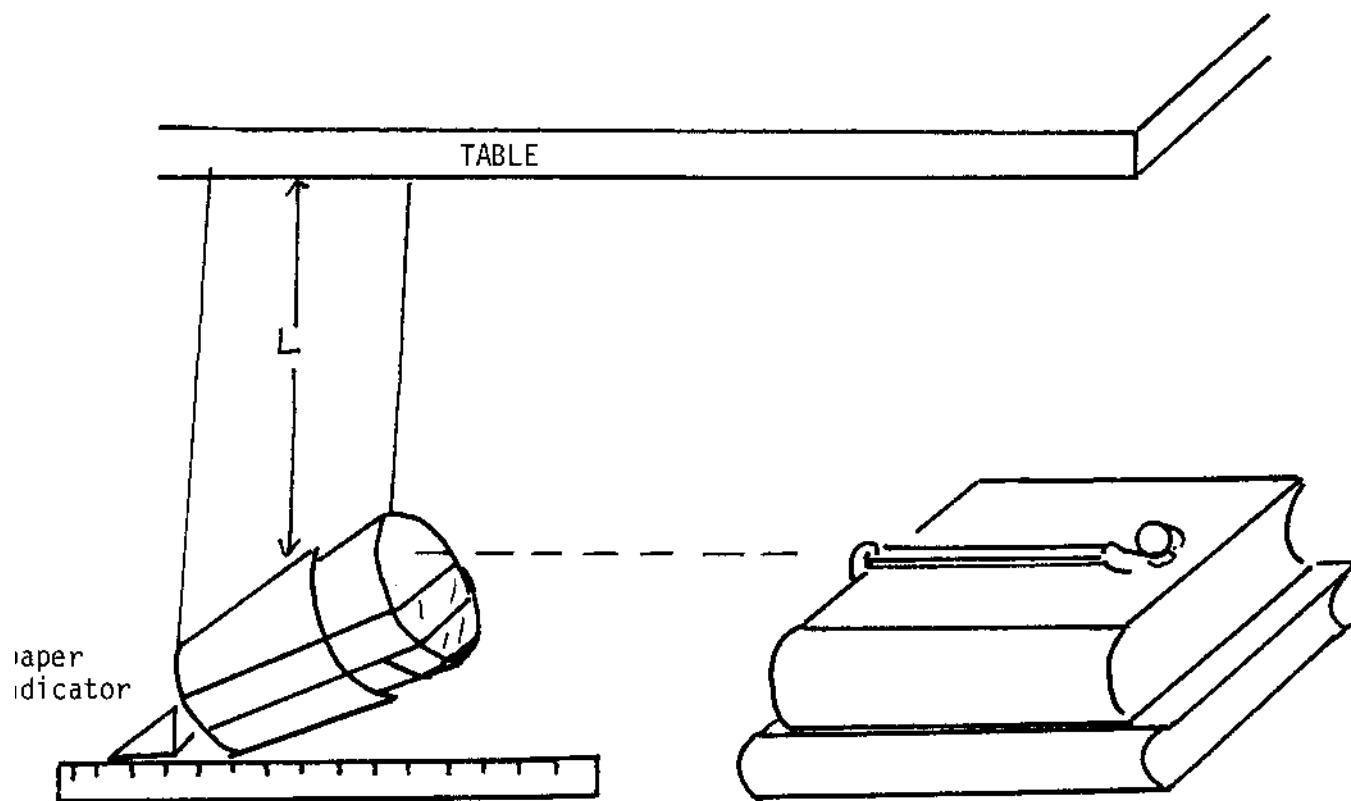
$$v_c = \ell \sqrt{\frac{g}{L}}$$

The initial velocity of the projectile, one marble, with twenty five in the cup, is

$$v_p = 26\ell \sqrt{\frac{g}{L}}$$

It is told the famous physicist, Lord Kelvin, used to perform the experiment by shooting an old, large-bore rifle into a suspended block of wood. One day

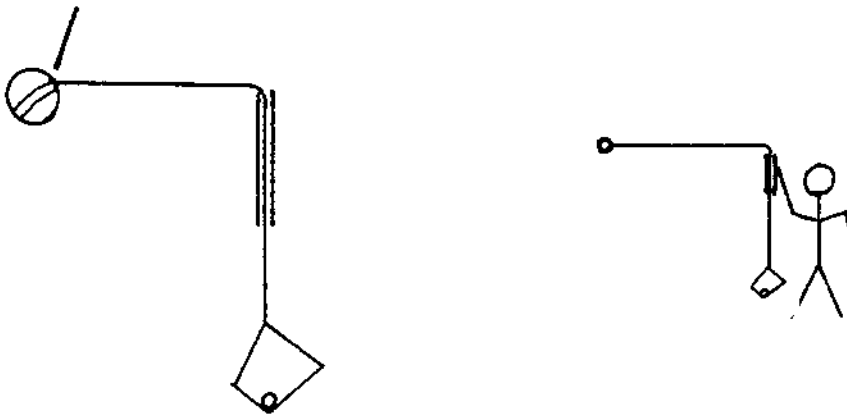
he missed, and the bullet travelled through the wall, almost hitting the lecturer next door. Kelvin dashed round to find the petrified lecturer, and the class crying, "You missed, you missed. Try again."



Experiment 1.32 Central Forces

Materials: - Soda straw, string, several marbles, sticky tape, cup

Attach a marble at one end of the string using tape.



Pass the string through a short section of soda straw, and attach a foam or paper cup at the other end, leaving about 4 feet of string between.

Now whirl the marble around your head holding the straw. Place five marbles in the cup.

The force pulling the rotating marble inward is provided by gravity acting on the marbles in the cup. This is a constant force. By adjusting the speed of rotation, keep the marble at a fixed short distance from the center, and then allow it to move outwards, again holding it at constant speed. Put several marbles in the cup as a counterweight, and repeat the experiment. You will find it is easier to allow the marble to move outwards-friction raises problems in returning to the center. What do you learn?

Qualitatively: -

The farther out the marble is, the slower you need to swing it around, to stop the counterweight pulling it inwards, so fewer rotations per second are needed to balance the counterweight. Putting more marbles in the cup makes you swing the marble round faster if you don't want it pulled inwards.

Quantitative: - Make a mark at a fixed distance, say three feet, from the marble, using a marking pen, and count the number of rotations per

second, with, say, five marbles in the paper cup, to hold the mark just above the straw. The acceleration for an object moving uniformly in a circle is given by $\text{acceleration} = \frac{v^2}{r}$ where v is the velocity of the object, and r is the radius of the circle in which it moves. The force providing this acceleration is supplied by the weight of the five marbles, so we have

$$5 \text{ mg} = \frac{mv^2}{r}$$

$$\text{or } r = \frac{v^2}{5g}$$

Suppose you count n rotations per second. (count for 20 seconds and divide by twenty). We may calculate n as follows $v = 2\pi rn$ then

$$r = 3 = \frac{(2\pi \times 3 \times n)^2}{5 \times 32} \quad \text{or } n = 1.12 \text{ revs/sec.}$$

You can easily check to find if this formula is correct by timing the rotations with difference radii of swing. You should find

$$g = \frac{(2\pi rn)^2}{Nr} \quad \text{ie. } n = \sqrt{Nrg}/2\pi r$$

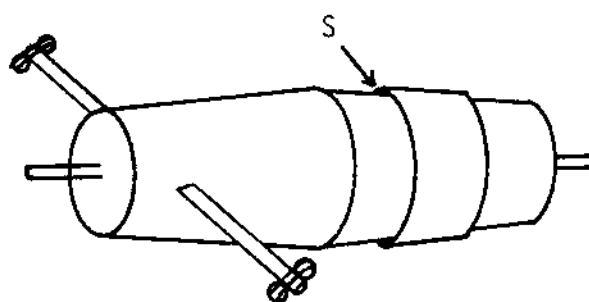
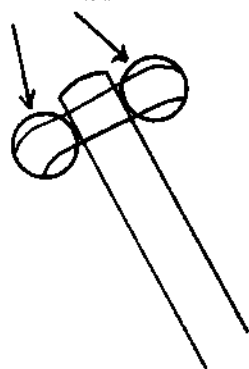
when N is the number of marbles in the cup
 g is the acceleration due to gravity (9.81 m/s^2)
 n is the number of rotations per second
 r is the radius of swing

Experiment 1.33 Moment of Inertia

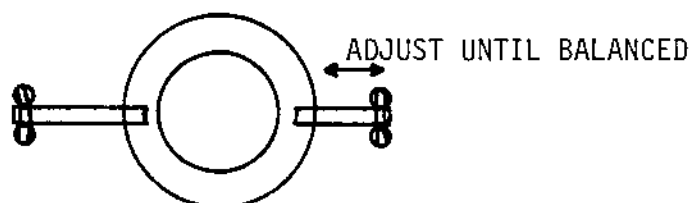
Materials required: - pulley, straw, marbles

Procedure: - The experiment is to investigate the behavior of a rotational system set spinning by a weight. Make holes through the cup forming part of the "pulley" discussed in experiment 29, and push a straw through as shown. Attach two marbles at each end

TWO MARBLES



with tape, then adjust the straw back and forth until it balances.



Fasten a string with tape to s, wrap around the groove, then hang a cup from the end. Time how long it takes the cup to reach to the floor with a marble in it.

Qualitative Question:

Explain why the dumbbell shaped device slows the acceleration of the cup, and the falling weights.

Quantitative Question:

If m is the mass of a marble, the moment of inertia of the dumbbell about the center is $4 m \left\{ \frac{\ell}{2} \right\}^2$ when ℓ is the length of the straw (generally 8 inches = 20.32 cm).

With one marble in the cup, the couple is

$$mgr = m \times 981 \times 3.8$$

So the total moment of inertia of the system is

$$I = 4 m (10.16)^2 + m \times 3.82$$

$$\text{Couple} = I \ddot{\theta} \quad (\ddot{\theta} = \text{angular acceleration})$$

$$\text{so } \ddot{\theta} = \frac{\text{couple}}{I} = \frac{981 \times 3.8}{4(10.16)^2 + 3.82} = 8.723 \text{ rads/sec}^2$$

To fall two feet the pulley rotates through an angle

$$\theta = \frac{24 \times 2.56}{3.8} = 16 \text{ rads}$$

in time t

$$\text{where } \theta = 1/2 \ddot{\theta} t^2$$

$$t = \sqrt{\frac{2 \times 16}{8.723}} = \underline{\underline{1.91 \text{ sec}}}$$

What do you get?

Calculate the radius of gyration from the time it takes for the weight to fall.

Experiment 1.34. Moments of inertia of books and straws

Equipment: paper clips, straws, book tape

Method: The concept of inertial mass is often difficult to express. Generally I do it by talking about the difference between kicking a football and a rock of the same size. The resistance to motion of the rock is obviously greater. The moment of inertia is also difficult to conceptualize. A simple way to demonstrate moment of inertia uses two soda straws. Put a paper clip in each end of one, then straighten out the wide end of two other paper clips, and push the narrow ends into the middle of the second straw, using a third opened-up clip. Hold the straw between the thumb and forefinger, as shown in Fig. 1, and rotate it to and fro. There is very little resistance to rotation with the second one. Do the same with the first straw, and you will feel a strong resistance to rotation. The mass of the two straws is the same, but the one with the clips in the middle has a much smaller moment of inertia.

A good quantitative experiment measuring the moment of inertia of a book requires, in addition, only sticky tape and a watch. Tape the book closed (Fig. 2a). Now, support it by a length of sticky tape about 9 in. long from the edge of the table as shown in Fig. 2b. Start it into a twisting oscillation about the vertical axis of the tape and measure the period by timing ten oscillations. Do the same from the second edge (Fig. 2c). To measure it about an axis perpendicular to the others through the center, pass a piece of tape around the book as shown in Fig. 3. Measure the period about the third axis. If the same length of tape is used for the support, and the mass of the book is fixed, the restoring torque for a given angle of rotation is constant. This means the square of the period is proportional to the moment of inertia. If the length of the sides of the book are a and b , the ratio of the periods about the two edges and the center should be as

$$a : b : \sqrt{a^2 + b^2}$$

since the book may be regarded as a rectangular sheet. Results accurate to about 5% generally ensue from this experiment.

Using 3/4 in. tape, we can assume approximately that the book is supported by the two edges of the tape as shown in Fig. 4. The restoring torque as the book is twisted is provided by the fact that as the tape turns, the book is lifted. If the book is rotated by θ , the restoring torque is Fd . F is the horizontal component of the force acting along the edge of the twisted tape. Since the total force acting along each edge is approximately $Mg/2$, this component is $(Mg/2) \sin \phi \approx Mg\phi/2$. Geometrically we have $\theta d/2 = \phi \ell$.

Thus: $F = (Mg/2) \phi = (Mg/2) (d/2\ell)\theta = Mg\theta d/4\ell$

The equation for the rotational simple harmonic motion is

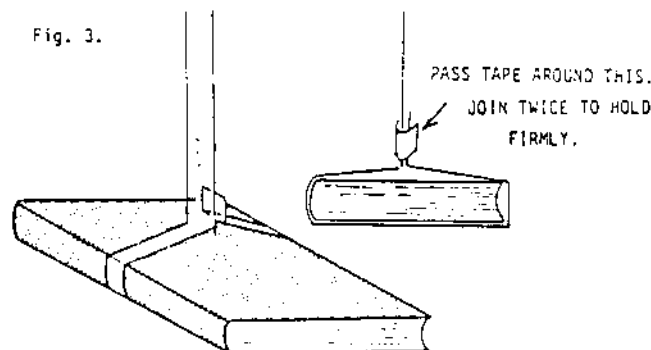
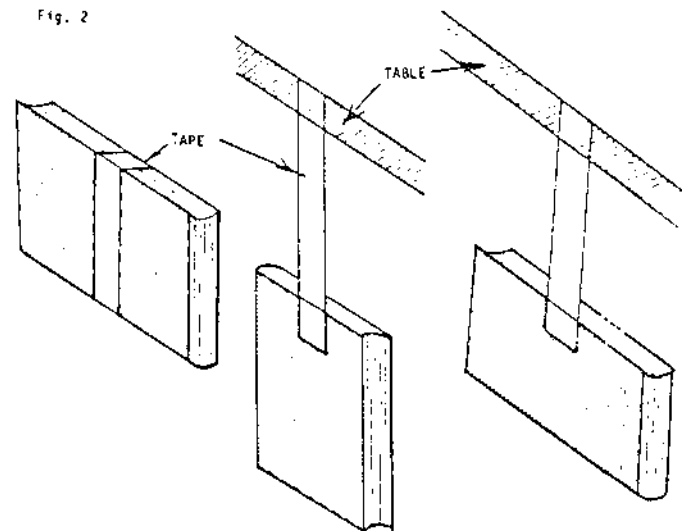
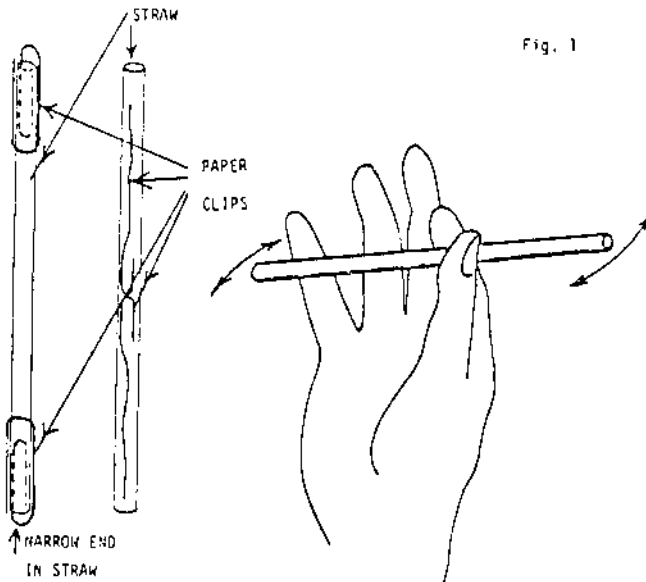
$$I d^2\theta/dt^2 = Mg\theta d^2/4\ell$$

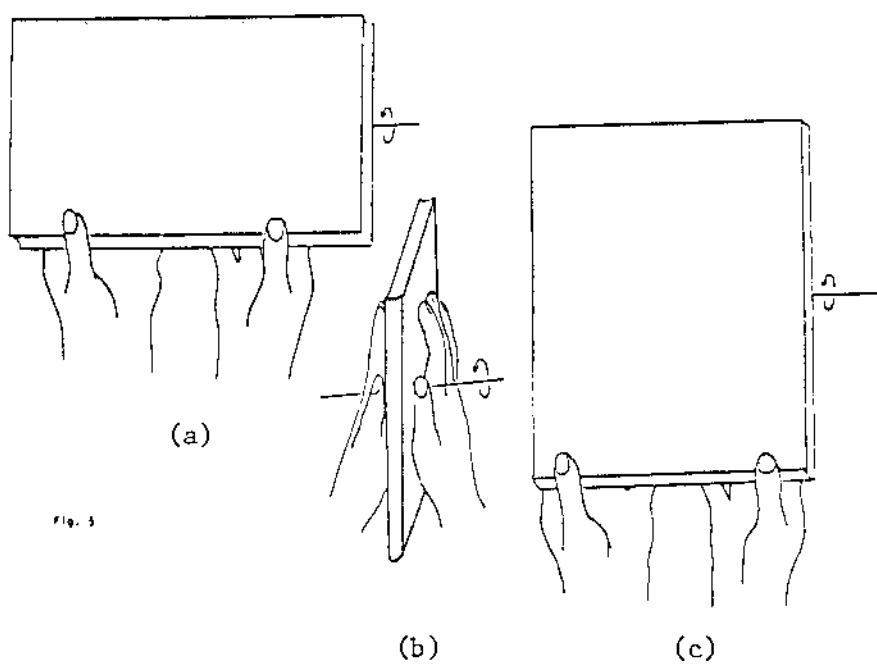
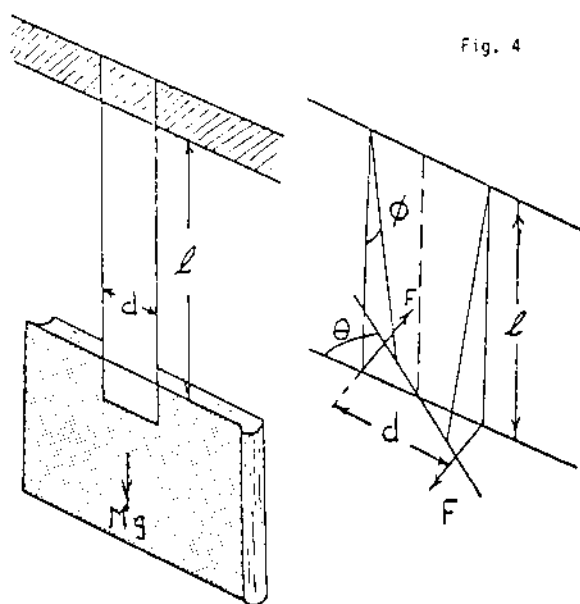
leading to a period $T = 2\pi \sqrt{4\ell I/Mgd^2}$

Now, $I = Mp^2$, where p is the radius of gyration,

$$\text{so } T = 2\pi \sqrt{4\ell p^2/gd^2} = 2\pi p \sqrt{4\ell/gd^2}$$

By measuring T , you can find p and hence calculate I . After you have measured the moment of inertia of the book, you can try an interesting experiment on stability. This works best with a book whose long and short sides differ considerably. With the book taped shut toss it in the air so that it rotates about the long axis as shown in Fig. 5a. It will spin quite stably. Now hold it between two palms and spin it into the air (Fig. 5b). Again, it will spin stably. Lastly, toss it to spin parallel to the short side of the book (Fig. 5c). This proves impossible. Instead, the book twists and turns about no one axis. Whereas the first two axes are ones of minimum and maximum moment of inertia, and hence, stable, the third is not. It seems so intuitively obvious that it will spin stably about the third axis that it is very easy to win bets with this trick.

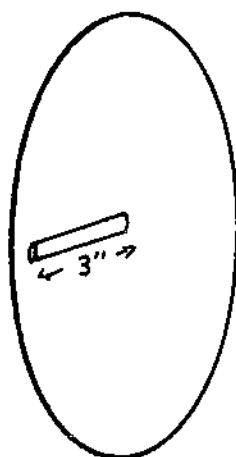




Experiment 1.35 Moments of Inertia

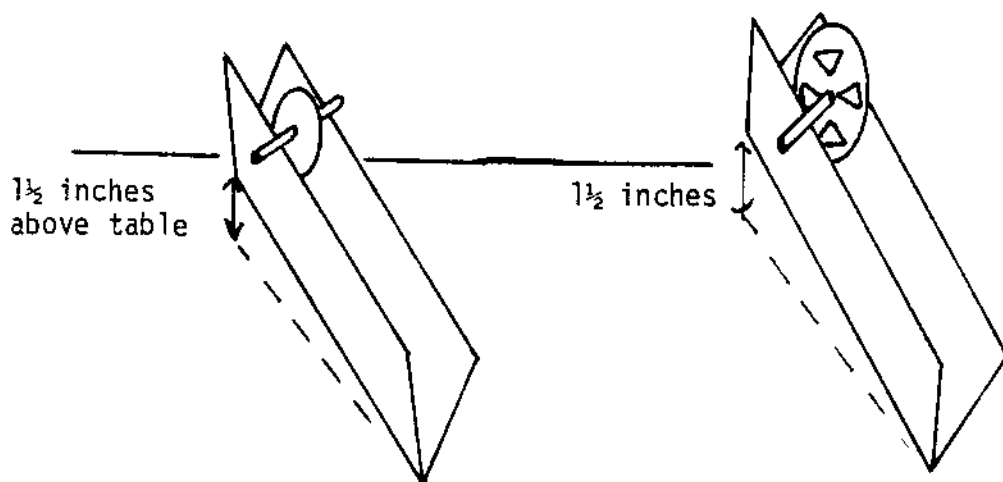
Materials: - Cut outs provided below. Straw.

Procedure - Cut out the two outlines on the next page. If you wish, you can paste them on card to make them stiffer. They should balance on the 0 which can be punched out with a pencil point so that a length of straw about six inches long may be passed half way through as shown below



Now, fold two $8\frac{1}{2}$ by 11 sheets and tilt them at the same angle (one end should be about $1\frac{1}{2}$ " above the other). Let the two figures run down.

Qualitative: Which gets to the bottom first? Both cut outs have the same area, and therefore the same mass, but the mass is more spread out in one case than the other, so that the moment of inertia is about twice in one case what it is in the other. The one with the larger moment of inertia accelerates more slowly.



Quantitative Questions: - The moment of inertia of one disc is twice the other, and the masses are the same. Hence, the force is the same, and the couple spinning them also

$$C = I\ddot{\theta}$$

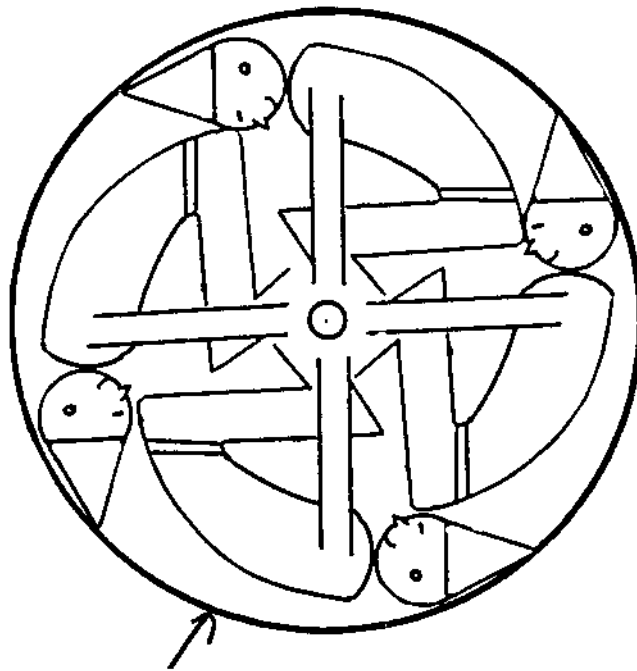
$$\text{Hence } \theta = \frac{1}{2} \frac{C}{I} t^2$$

$$C = \text{couple}$$

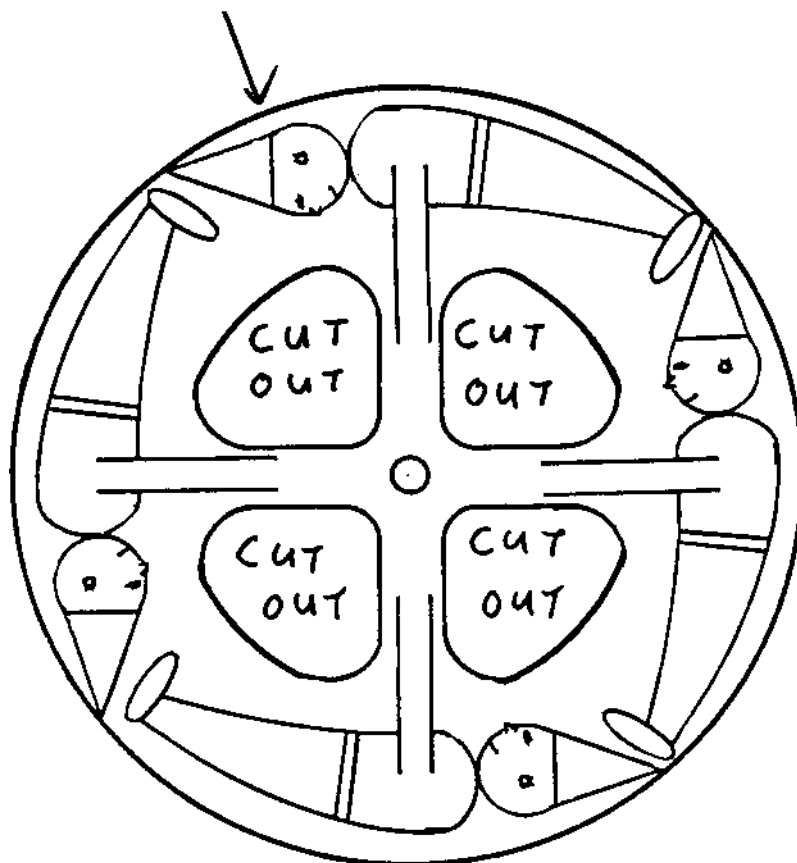
$$I = \text{moment of inertia}$$

$$\theta = \text{angular}$$

If one disc reaches the bottom in time t , the other disc, having rotated through half the angle, should only be half way down. How far down was it?



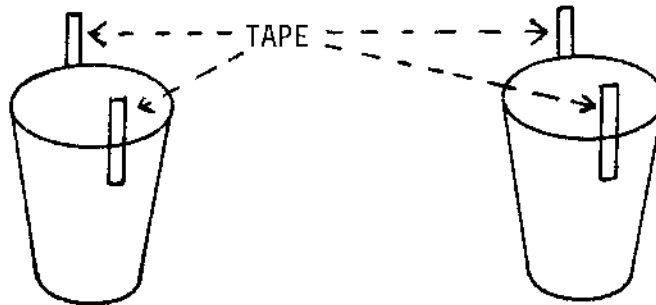
CUT OUT THESE TWO FIGURES



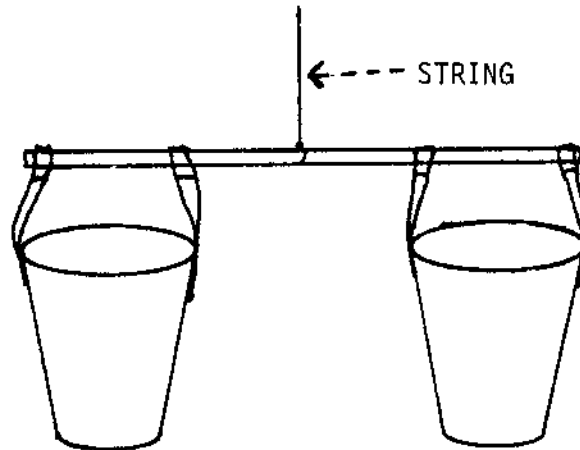
EXPERIMENT 1.36 Rotational Energy and Momentum

Materials required: cups, straw, string, ruler, about twenty marbles, books to prop up ruler, sticky tape.

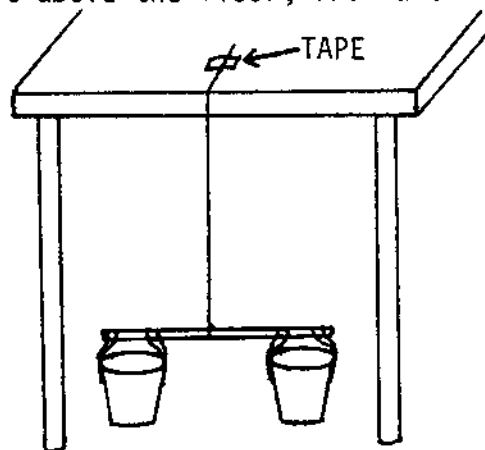
Procedure: Attach a piece of tape about 4" long to each side of two cups.



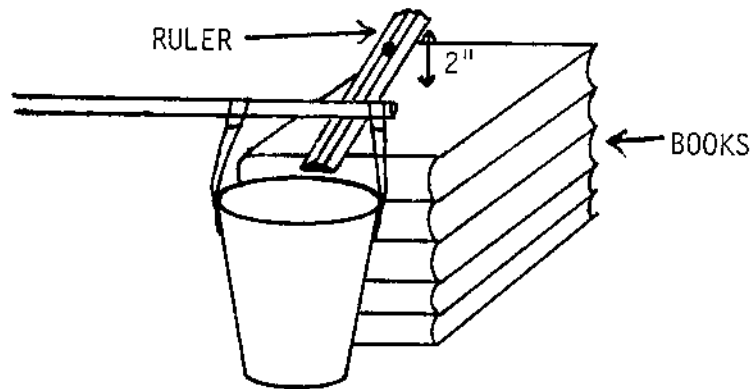
Wrap the tape over a straw, so that the cups are suspended at the ends as shown.



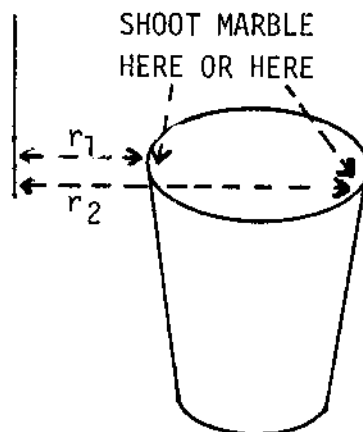
Now, tie a piece of string to the center of the straw, place ten marbles in each cup, and suspend just above the floor, from a table or chair back.



Allow the system to come to rest. Support a ruler on a pile of books so a marble can run down into a cup.

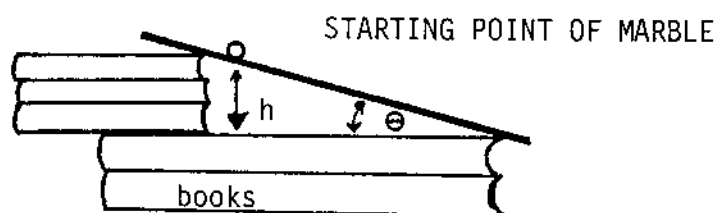


Let the marble roll down a known height, into the cup. Time half a rotation, and also measure the angle of rotation (number of turns) until the cups stop spinning. Do this twice from the same height, allowing the marble to enter the cup at the two edges, as shown. These are about $2\frac{1}{2}$ inches apart, so the arm of the rotational impulse can be varied from about 4" down to $1\frac{3}{4}$ ".



Qualitative: Why is the rotation slower and the angle of rotation smaller when the marble enters near the support string, even though the energy delivered is the same? Where does the excess energy go?

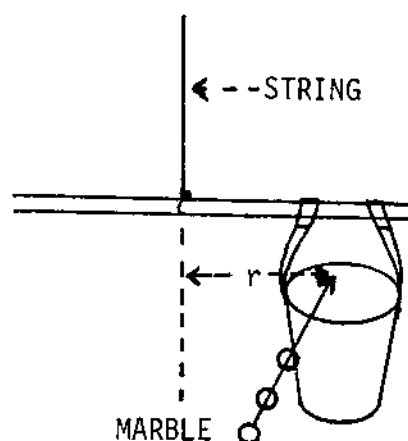
Quantitative: The kinetic energy of the impinging marble is $\frac{1}{2}mv^2 = mgh$ where v = velocity leaving the chute.



(more accurately $\sqrt{2gh} \cos \theta$ is the horizontal component of velocity). So the momentum

$$mv = m \sqrt{2gh} \cos \theta$$

and the moment of momentum = $mr \sqrt{2gh}$



so $I\dot{\theta} = mr \sqrt{2gh}$ where $\dot{\theta}$ is the angular velocity, $d\theta/dt$

$\dot{\theta}$ is obtained from the time T for half a revolution.

$$\dot{\theta} = \frac{\pi}{T}$$

Hence I is obtained in terms of m . Is this roughly equal to (mass. of marbles) $\times$ (distance from center)²? The kinetic energy of rotation is converted to torsional energy of the string. If ϕ = angle of rotation of the string,

$$\frac{1}{2} I \dot{\theta}^2 = \frac{1}{2} k \phi^2$$

Where k is the torsional constant for the string
i.e. couple = $k\phi$

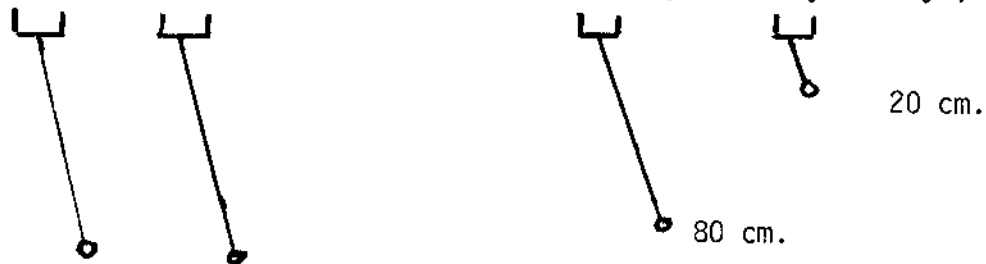
Hence, ϕ should be proportional to $\dot{\theta}$, and also proportional to the value of r (shown in the figure) if we release the marble from the same height each time.

Is this true for the two cases where the marble enters at opposite sides of the cup?

Experiment 1.37 Time and the pendulum

Materials: Marbles, string.

Procedure: Ever since Galileo (1564-1642) the pendulum has been associated with time. Tape a marble to the end of a string about four feet long. Make a second pendulum the same way. Let us see how Galileo was able to deduce the laws of the pendulum before clocks were available. Make both pendulums the same length, and hang from a suitable support - anything will do, as a last resort, use your hand (it is, however, not very steady!)



Now pull back both pendulums so that one swings through a larger arc than the other (a larger amplitude). Release them at the same time. Do they keep time, independent of the arc? They should, since the period (time of one oscillation, to and fro) of a pendulum is independent of the amplitude (if it is small!)

Now attach two marbles to one pendulum bob, and repeat the experiment. Is the period dependent on the masses of the bob?

Lastly, make one string four times the length of the other, say, 9 inches and 36 inches. How many swings of one bob equal that of the other? Make one length nine times the other--say 15 cm. and 135 cm. Again, how many swings of one bob occur for one swing of the other?

The time of one oscillation, T , the period, depends on the length of the pendulum l . Is the period proportioned to l i.e. double the length, double the period? Is $T^2 \propto l$ i.e., double the period, four times the length or $T^3 \propto l$ double the period, eight times the length?

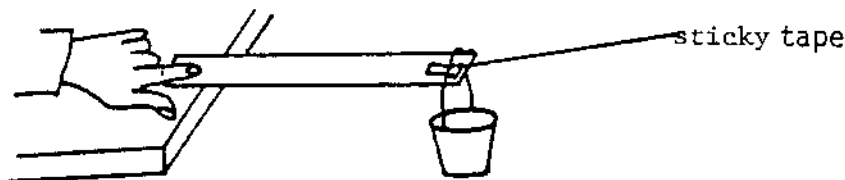
Is the period constant for very large arcs, say 70° each way? It is not, and, in fact the period is more accurately given by

$$T = T_0 \left(1 + \frac{\theta^2}{16} \right) \text{ where } \theta \text{ is the amplitude in radians.}$$

Experiment 1.38 Oscillations of a spring -- Inertial and Gravitational Mass

Materials: ruler, sticktape, marbles, string and a paper or foam cup

Procedure: Pass a string through the top of the cup and attach it to the end of the ruler, as shown. Hold two inches of the other end of the ruler as



tightly as possible to the table top. Place ten marbles in the cup. Flick the far end of the ruler, and note how the vibrations produced gradually die away. One oscillation is one complete up-and-down motion at the ruler end. Count the number of oscillations in ten seconds, or five seconds if the oscillations die away too quickly. The oscillations are very rapid with few marbles, and the end of the ruler must be examined very closely to count them. If difficulty is experienced, use two rulers taped tightly together (making it about two feet long), or a yardstick or meter ruler, if available. You may use a watch to time the oscillations, but if it lacks a second hand, tape a marble to a piece of string, and have one student hold it 39 inches (99.4 cm) above the marble. Each swing (two swings to one oscillation) takes one second, so one observer calls out at the beginning and end, after ten seconds, and the other counts the number of oscillations of the ruler.

Put fifteen, twenty, and then, if possible, twenty-five marbles in the cup and again count the number of vibrations in ten seconds.

Now count the number of oscillations in five seconds for very large vibrations when you pull the ruler way down before releasing (large amplitude) and for tiny vibrations, where you barely touch the ruler to set it going. Lastly, shorten by two inches the length of ruler vibrating, (i.e. hold four inches of ruler against the table) and again count the number of oscillations in ten seconds.

Qualitative Questions: Note how the oscillations become slower, the more mass is at the end of the ruler, and become faster, the shorter the ruler, (the stronger the springiness).

Does increasing the amplitude of the vibrations make them faster or slower?

Quantitative Questions: The frequency f is the number of oscillations per second - i.e. the number you counted divided by five or ten. The period T (the time of one oscillation) is

$$\frac{1}{f}$$

Theory shows that $T = 2\pi \sqrt{\frac{m}{k}}$

where m is the mass on the spring, and k is the spring constant. Plot on the graph paper on the following page the period T against the square root of the mass on the spring (the number of marbles). The dots on the graph are typical examples. In this experiment, the ruler is the spring.

no. of marbles	square root of number of marbles	number of oscillations in five seconds n	frequency f (n/5) oscillations per second	period 1/f seconds
(5)	(2.2361)			
10	3.1622			
15	3.873			
20	4.472			
25	5			

Is the result a straight line?

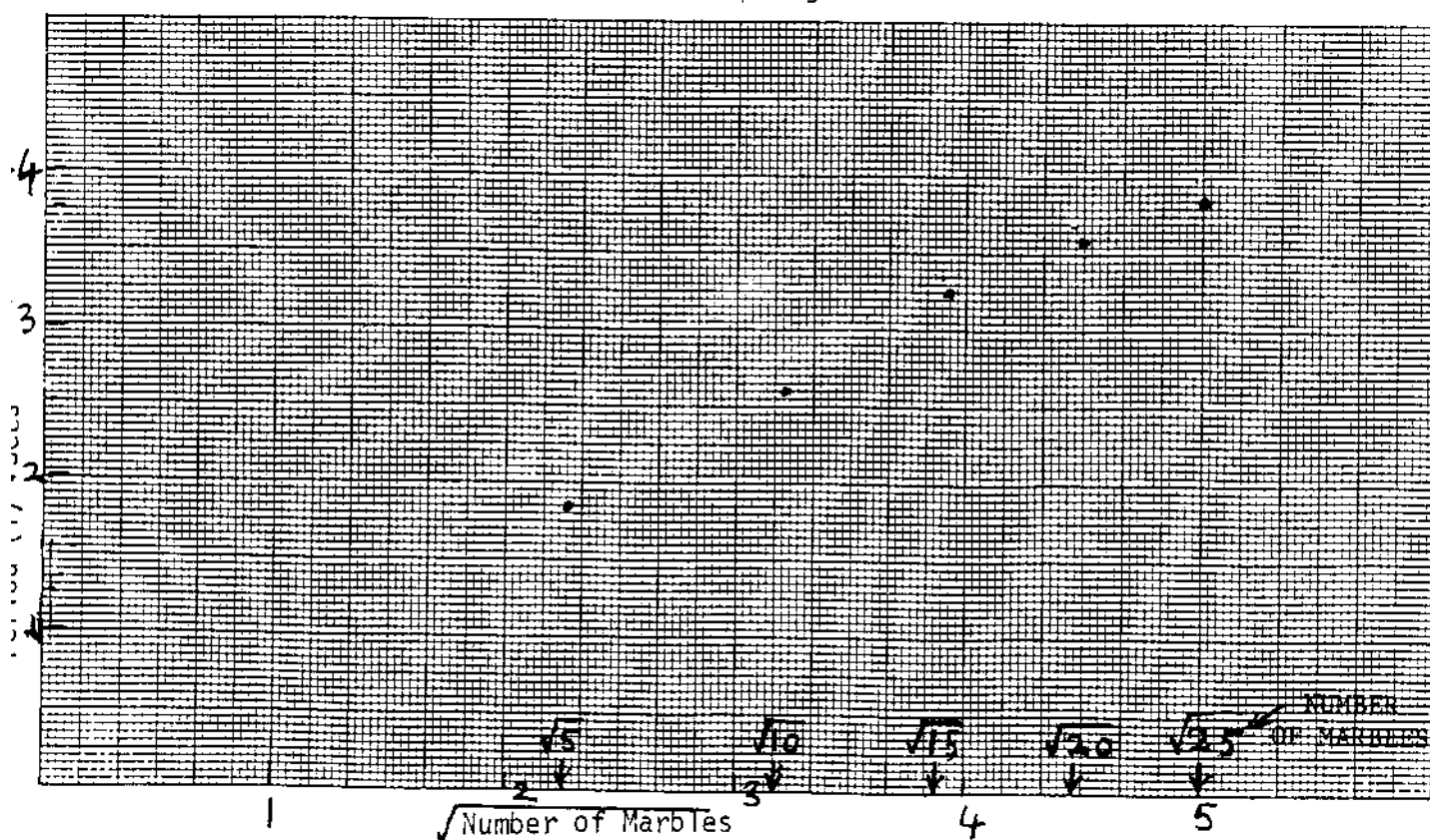
What does this show?

Draw a line through the points.

Does it go through the origin (no mass, $T = 0$)?

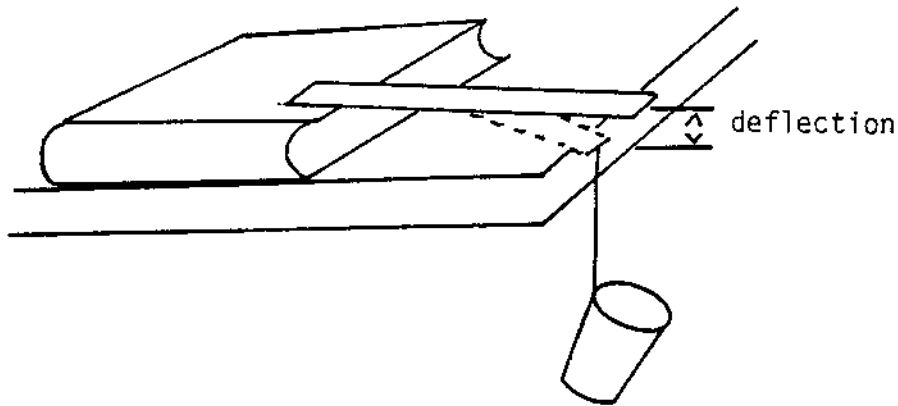
As a further exercise, you can check whether $T^2 \propto 1/k$. You measured T for two different values of k (lengths of ruler). Use a lot of marbles in the cup. Now you need to find k , which comes from the equation

$$\text{force} = k \times \text{extension of spring}$$



To find the ratios of the two values of k , hang a weight on the end, and notice how much it deflects the end of the ruler with first 10 cm. and then 20 cm. of the ruler extending over the edge of the table to which it is held.

$$\frac{k_2}{k_1} = \frac{\text{deflection with 10 cm.}}{\text{deflection with 20 cm.}} \quad (k \propto 1/\text{deflection})$$



The ratio of the deflections is the ratio of the values of k . See if this ratio is the same as the ratio of the two values for T^2 obtained with two and four inches of the ruler pressed to the support.

Notice that we can obtain the ratio of inertial and gravitational masses, since $k = \frac{mg}{\text{deflection}}$ where here m is the gravitational mass and

$$T = 2\pi \sqrt{\frac{m}{k}} \quad \text{where in this case } m \text{ is the inertial mass.}$$

This ratio is invariably one.

An interesting extension of this experiment shows students the relationship of musical pitch to frequency.

Take the cup away, and examine the vibration as that portion of the ruler vibrating over the book (or the edge of the table to which it is held) is shortened. With the 25 cm of the ruler extending over the table edge, it is easy to observe the ruler vibrating, and it may even be possible to count the oscillations. As the extension is shortened, the vibrations clearly increase in frequency, until a note can be heard. The frequency can be determined by adjusting until it is in unison with a soda straw 18.3 cm long which vibrates if blown* at 900 Hz. This shows high frequency, high pitch and a shortend ruler go together. The restoring force increases as the ruler is shortened, as does the vibrating mass, so the frequency is a strong function of the length of the ruler - for the wooden ruler I was using, I find, if the period in seconds is T and the length of the ruler L cm

$$T = 0.0000417 L^{1.8}$$

* Experiment 7.03

EXPERIMENT 1.39: Coupled and Forced Oscillations

Materials Required: Marbles, string, tape, heavy weight (such as a drink can)

Coupled oscillations present one of the most intriguing problems in physics. The ramifications extend, on the one hand, from the quantum mechanical aspects of chemical bonding, and the shift in energy levels of coupled atomic systems, to the coupled torsional and bell type vibrations of the whole earth, which have a period of about an hour, excited by a volcanic eruption or earthquake. It is very simple to demonstrate coupled oscillations using four marbles and a piece of string. Use sticky tape to attach the marbles to four strings, two 30 cm. long, and two 15 cm. long. Then tie the strings to a third string stretched, not too tautly, between the backs of two chairs, as shown in fig. 1. The pendulums are all "coupled" together by this string. What happens when one long pendulum is set swinging? The small pendulums are unaffected, and the motion handed back and forth between the long pendulums - and of course, the reverse happens when a short pendulum is disturbed. The normal modes of the coupled system can then be demonstrated by holding the two long pendulums at the same distance transverse to the string between the chairs, and releasing them together as shown in fig. 2. They continue to swing with a constant period until the motion damps out. For the other normal mode, also shown in the figure, the two pendulums are held out an equal distance on opposite sides and released simultaneously. Again, there is no transferring the motion back and forth. Measure the period of the two normal modes. The mode when only one pendulum is released is a combination of equal parts of the two normal modes, which beat against one another, so the frequency difference of the two normal modes is the frequency at which the motion is transferred too and fro between the two pendulums. By moving the pendulums closer together, or apart on the string between the chairs, you can show that tight coupling (closer together) gives a shorter period for transfer of the motion, and weak coupling (farther apart) gives a longer period.

Forced oscillations can be demonstrated using the same system. One pendulum has its marble replaced by a heavy weight - a soft drink can (full) is convenient. The other pendulum can have three paper clips in place of the marble. These are more heavily damped, and the addition of a small piece of paper to the clip leads to even heavier damping. The arrangement shown in fig. 3 works well. The string to the clips passes through another clip taped to the cross string. The length of the light pendulum may then be easily changed by raising or lowering the clips, without affecting the heavy pendulum. Quantitatively then, the observations to be made are the amplitude and phase of the paper clips as the pendulum supporting them is lengthened. Fig. 4 shows these as a function of the length of the pendulum, for three paper clips, and three paper clips plus a piece of paper 5 cm square. The resonance is more highly damped in the latter case. The heavy pendulum was 60 cm long. At resonance, the phase lag of the clips is $\pi/2$, 90° , with the result that when the heavy weight is in the middle, the clips are at the end, and when the weight is at the end, the clips are in the middle. This device is a modification of Barton's pendulums, when not one, but many pendulums of different length are supported on the cross string, so that the amplitude and

phase can be compared simultaneously. Resonance occurs when the force has the same frequency as the undamped free oscillations, not the damped oscillations. The highly damped pendulum has a much smaller amplitude at resonance, but the resonance is broader.

An even simpler demonstration can be made with a piece of string and two weights, one heavy and one light. The heavy weight could be a rock, book, or soft drink can. The light weight can be three or four paper clips. The soft drink can can be supported by passing the string through the ring on top. Tie the string as close to the center of the top as possible and fasten the thread to the paper clips to the center of the bottom with sticky tape. This way, when the can spins it will not affect the oscillations. Suspend the heavy weight from a suitable support--a coat rack, table, etc. by a thread, or fine string about two feet long. I found it easiest to bend a paper clip into a hook, attach the string to it and hang the can from the top of a door way, as shown in fig. 5.

First start the paper clips swinging. The free oscillations on their own decay rapidly. This is a "transient". Now start the heavy weight swinging. This produces the periodic force, which causes the clips to oscillate, their amplitude slowly building up. When the string to the clips is short, they stay in phase with the heavy weight, and their amplitude of oscillation is small, relative to the can. As the second string is lengthened, the amplitude of the oscillations builds up, until at resonance, when the two strings are of approximately equal length, starting from rest the oscillations of the clip will build up to a swing about ten times wider than those of the can. This is a very effective demonstration. The clips are also obviously 90° out of phase with the can, being at the end when the can is in the middle, and vice versa, as shown in fig. 6. Lengthening the thread to the clips still further, causes the oscillations to die down, until when the thread to the clips is twice as long as the string to the can, the oscillations will appear as in fig. 7 with the clips approximately 180° out of phase to the can.

Quantitative Solution

The Quantitative part of this problem can get quite involved. It is first necessary to examine the solution to the equation for forced oscillations. The equation is

$$F_0 \cos \omega t = d^2\theta/dt^2 + \gamma d\theta/dt + \omega_0^2 \theta$$

where $\omega_0/2\pi$ is the free oscillation frequency of the paper clip, ($\omega_0 = \sqrt{g/l}$ where l is the length of the pendulum), γ is the damping constant per unit mass, F_0 is the amplitude and $\omega/2\pi$ the frequency of the applied force per unit mass (i.e. force/mass of the paperclip) and θ the angular displacement of the string to the paper clips. When $F_0 = 0$, the transient solution to this equation is

$$\theta = A \exp(-\gamma t/2) \cos(\sqrt{(\omega_0^2 - \gamma^2/4)} t - \psi)$$

where ψ is an arbitrary phase angle. The solution with the applied force present, ultimately tends to

$$\theta = A \cos(\omega t - \phi)$$

where

$$A = F_0 / \sqrt{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}$$

and

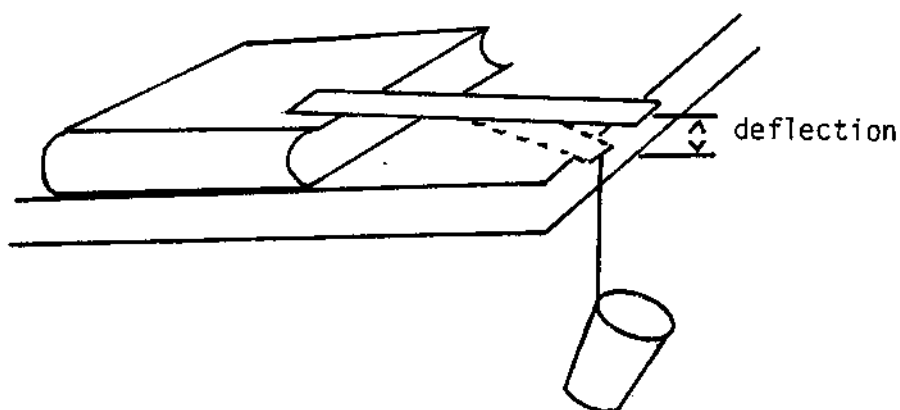
$$\tan \phi = \gamma \omega / (\omega_0^2 - \omega^2)$$

The significance of these two solutions is clear. The first is simply a decaying sinusoidal oscillation, and the second a steady oscillation to which the clips will ultimately settle down. However, if we start the can swinging with the clips initially at rest, we get a combination of two equations of the form

$$\theta = A \{ \cos(\omega t - \phi) - \exp(-\gamma t/2) \cos((\sqrt{\omega_0^2 - \gamma^2}) t - \phi) \}$$

To find γ , set the clip swinging alone, and note the time in seconds required for the amplitude to decay by a half, $T_{1/2}$. Since $\exp(-\gamma T_{1/2}/2) = 1/2$, $\gamma = 2 \log 2 / T_{1/2} = 1.386 / T_{1/2}$.

For three paper clips swinging from a 60 cm thread, $T_{1/2}$ turns out to be about 10 seconds and γ equals .139. If the can is also swinging from a 60 cm thread $\omega = 2\pi/T = \sqrt{g/L}$. T , the period is 1.55 seconds, giving $\omega = 4$ rad/sec. This also equals ω_0 at resonance. We now have all the required parameters, and the curve derived from these is the one shown in fig. 8c. 8a and 8b are the corresponding transient and continuous solutions. Now, try the experiment yourself--it only requires a few paper clips, string, and a soft drink can (actually, a can of beer was employed for these experiments.) Fig. 9 shows beats occurring when $\omega = 3.5$, rather than 4.0, ω_0 remaining the same. This is very noticeable experimentally. Damping reduces the effect of the beats.



The ratio of the deflections is the ratio of the values of k . See if this ratio is the same as the ratio of the two values for T^2 obtained with two and four inches of the ruler pressed to the support.

Notice that we can obtain the ratio of inertial and gravitational masses, since $k = \frac{mg}{\text{deflection}}$ where here m is the gravitational mass and

$$T = 2\pi \sqrt{\frac{m}{k}} \quad \text{where in this case } m \text{ is the inertial mass.}$$

This ratio is invariably one.

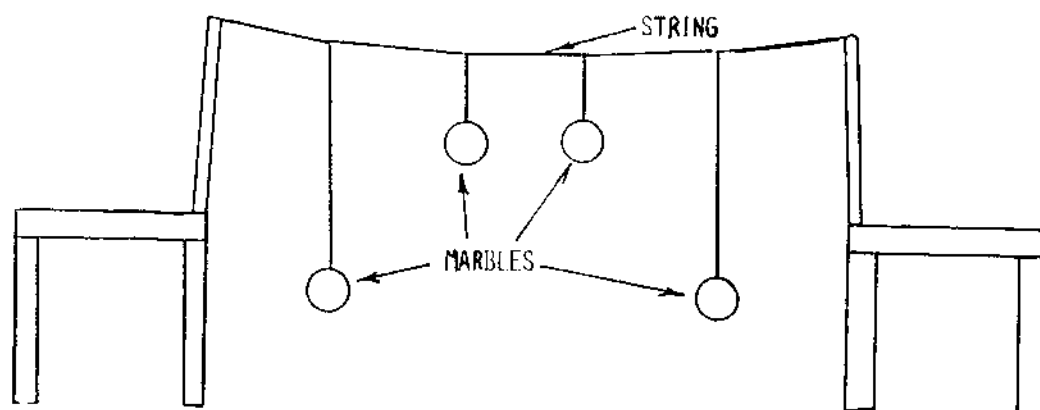


Fig. 1

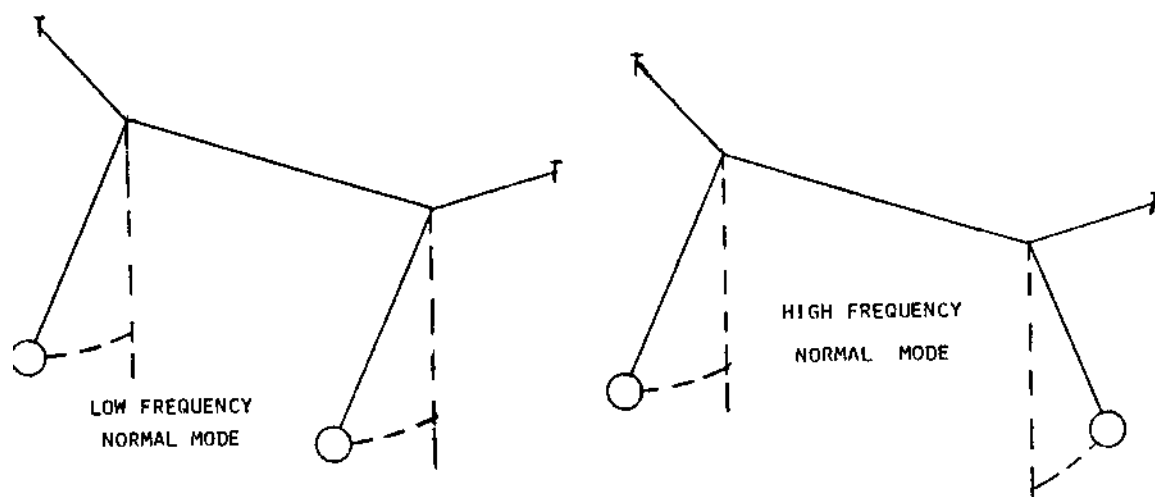


Fig. 2

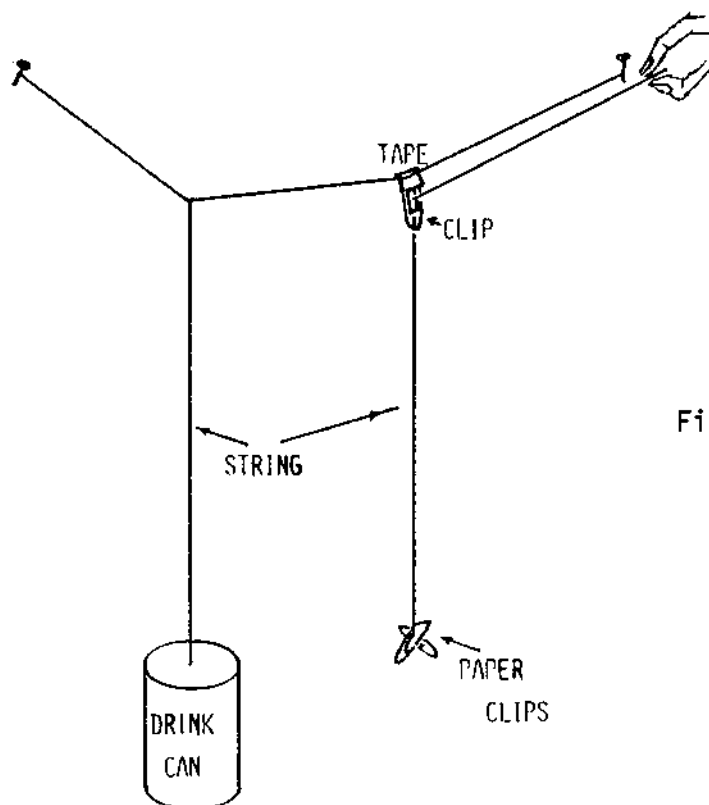


Fig. 3

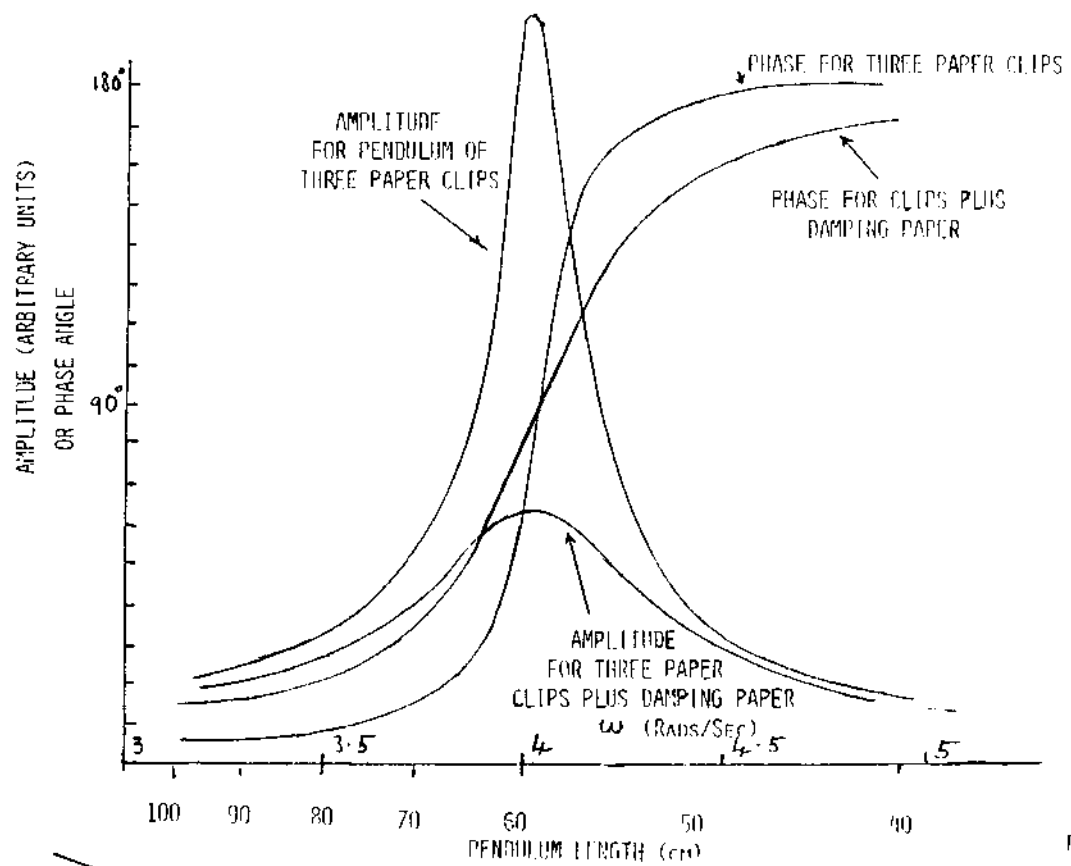


Fig. 4

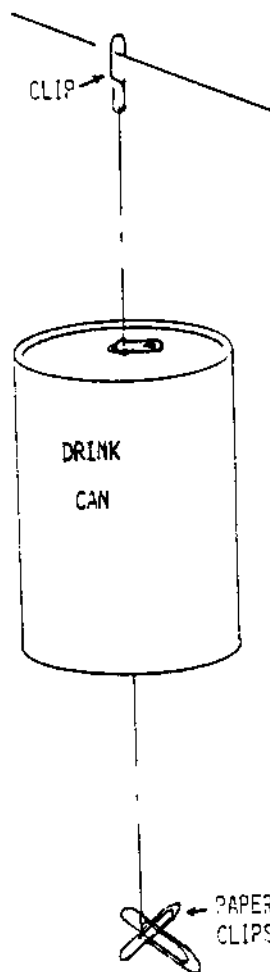


Fig. 5

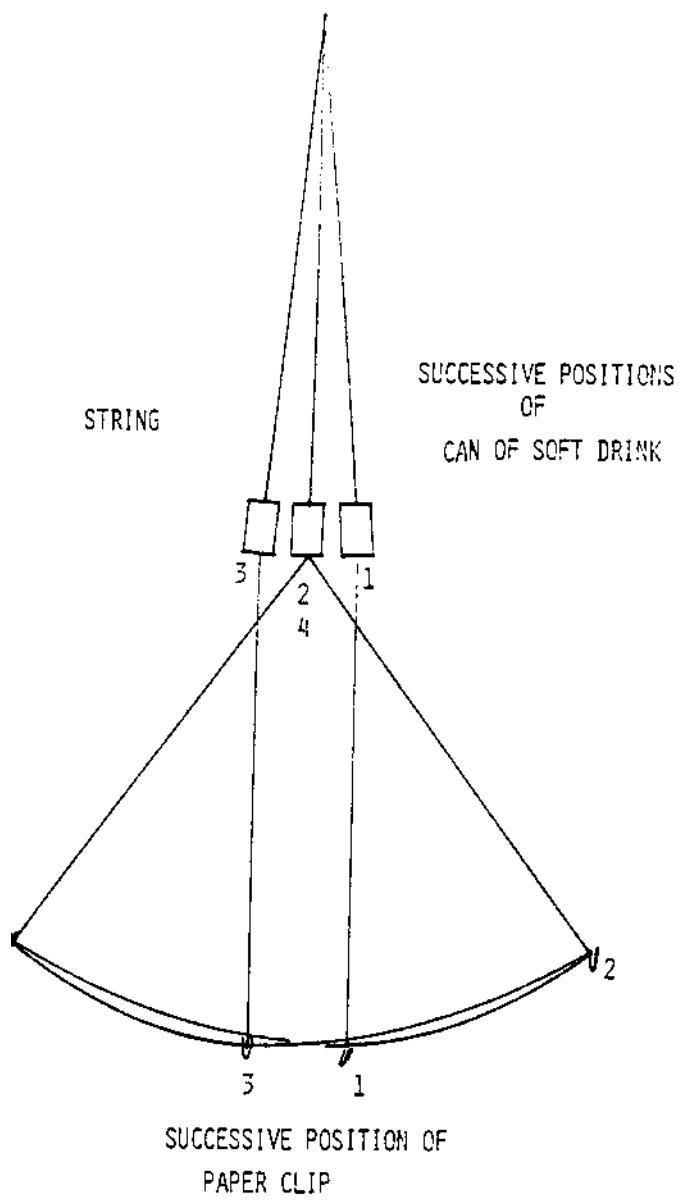


Fig. 6

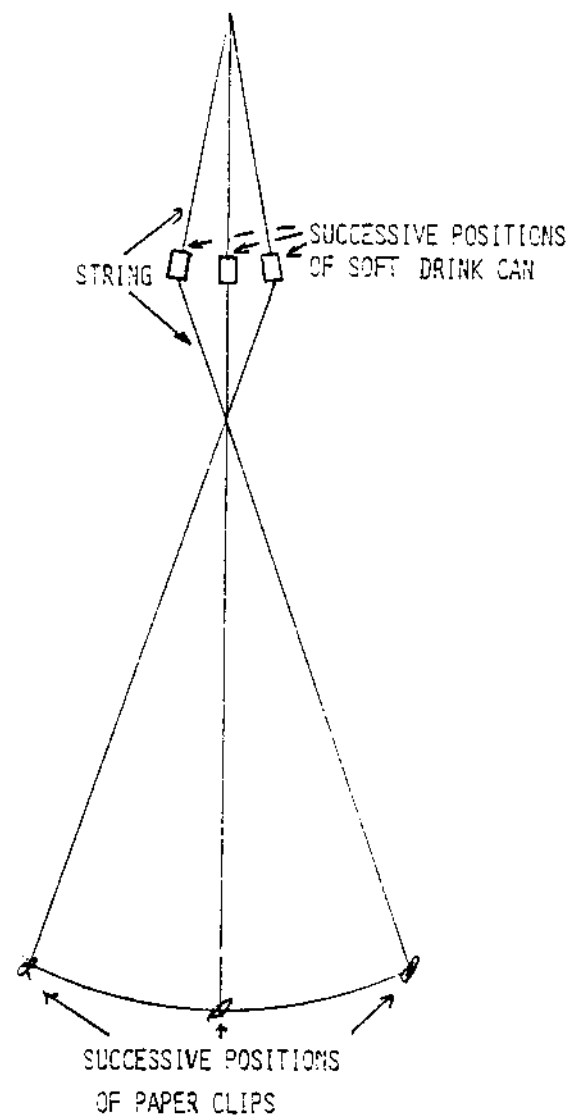
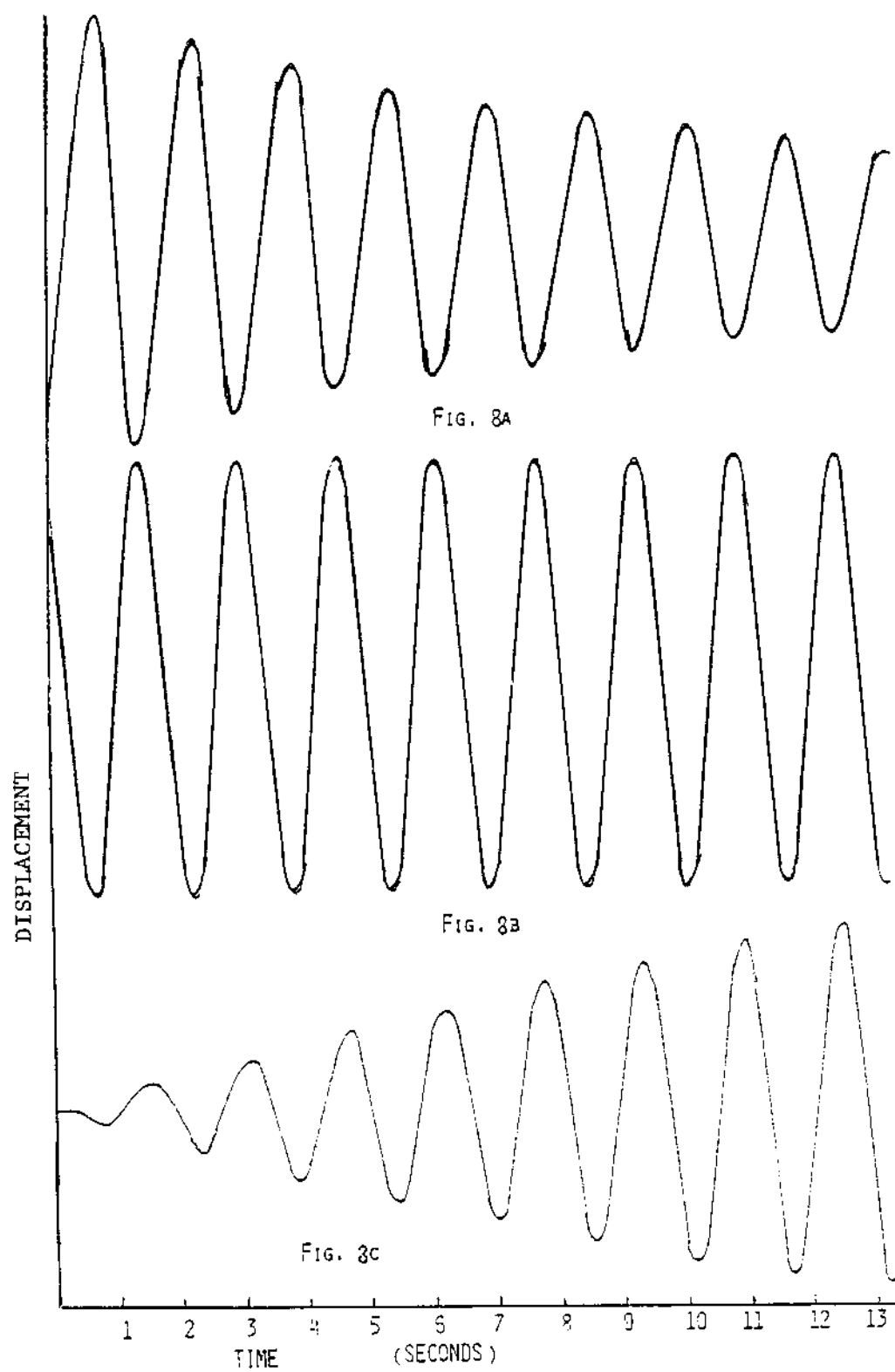


Fig. 7



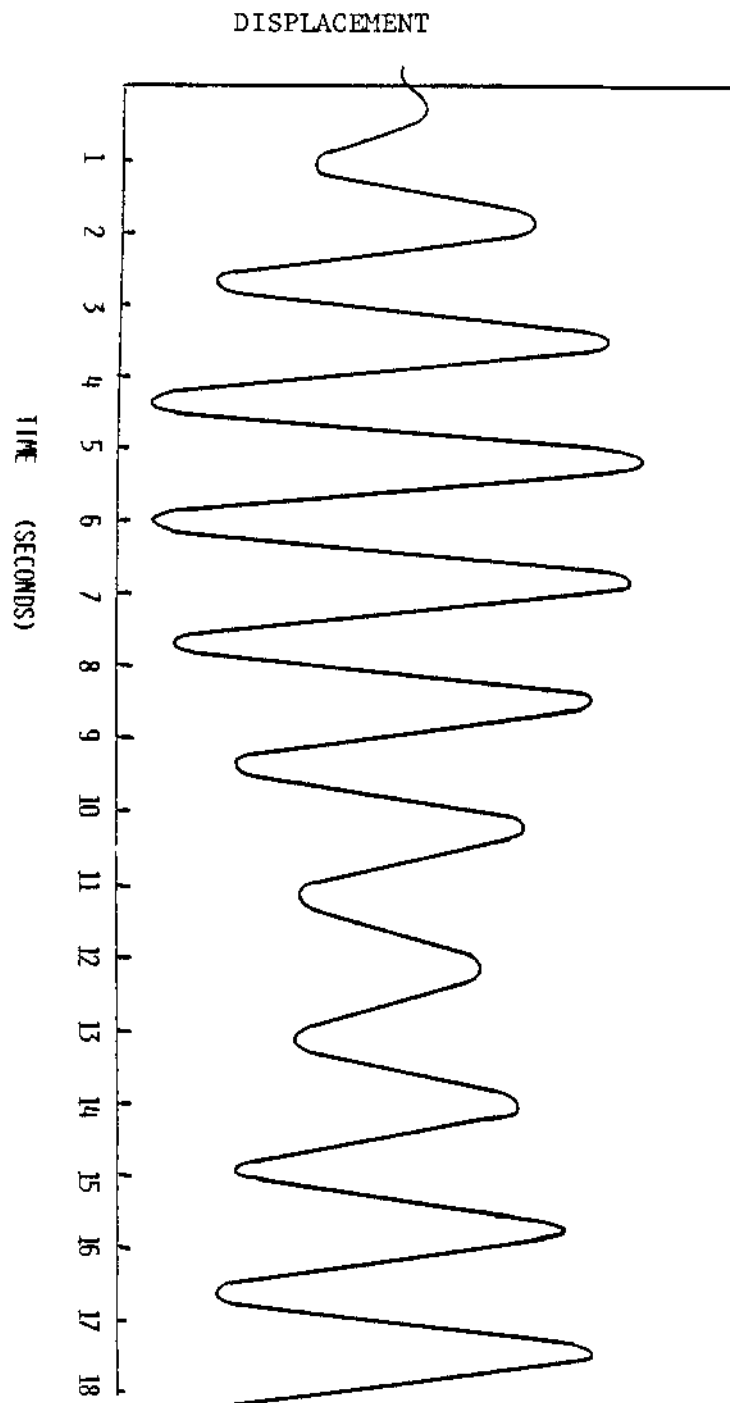
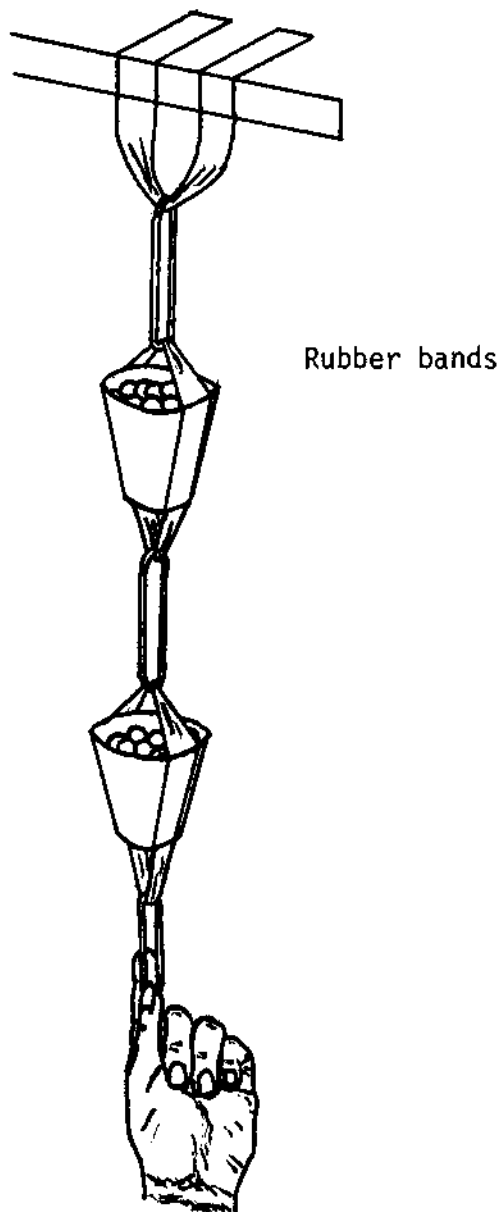


Fig. 9

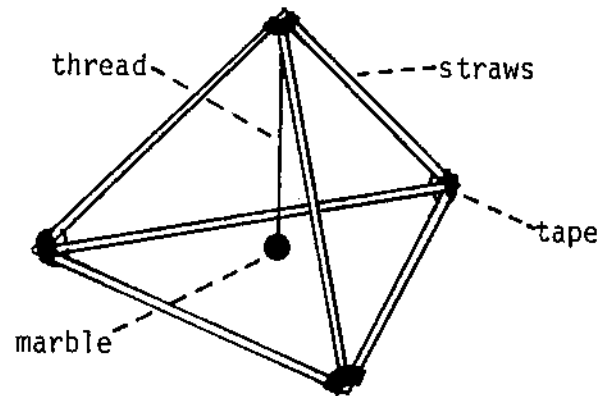
Another interesting coupled system is shown in the figure. A cup of marbles is supported from a table by a rubber band, and a second cup of marbles supported from this also by a rubber band, as shown. By applying a periodic force using a third rubber band, as shown, it is possible to set the system in oscillation. The modes are very similar to the pendulums, two normal modes, with the cup in or out of phase, in addition to which are complex modes composed of these two.



EXPERIMENT 1.40 Foucault Pendulum

Materials: Four straws, a marble, sticky tape and thread, or fine string.

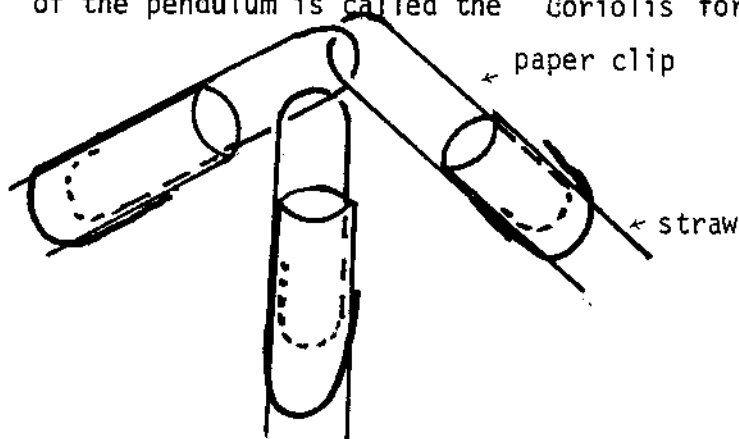
Procedure: Fasten the straws into a rigid tetrahedron, as shown, using tape. Attach a marble to a piece of thread using cello tape, then fasten it to the apex of the tetrahedron so it can swing freely. Better, the ends of the straws may be attached together using paper clips as shown below, and the thread held by a clip.

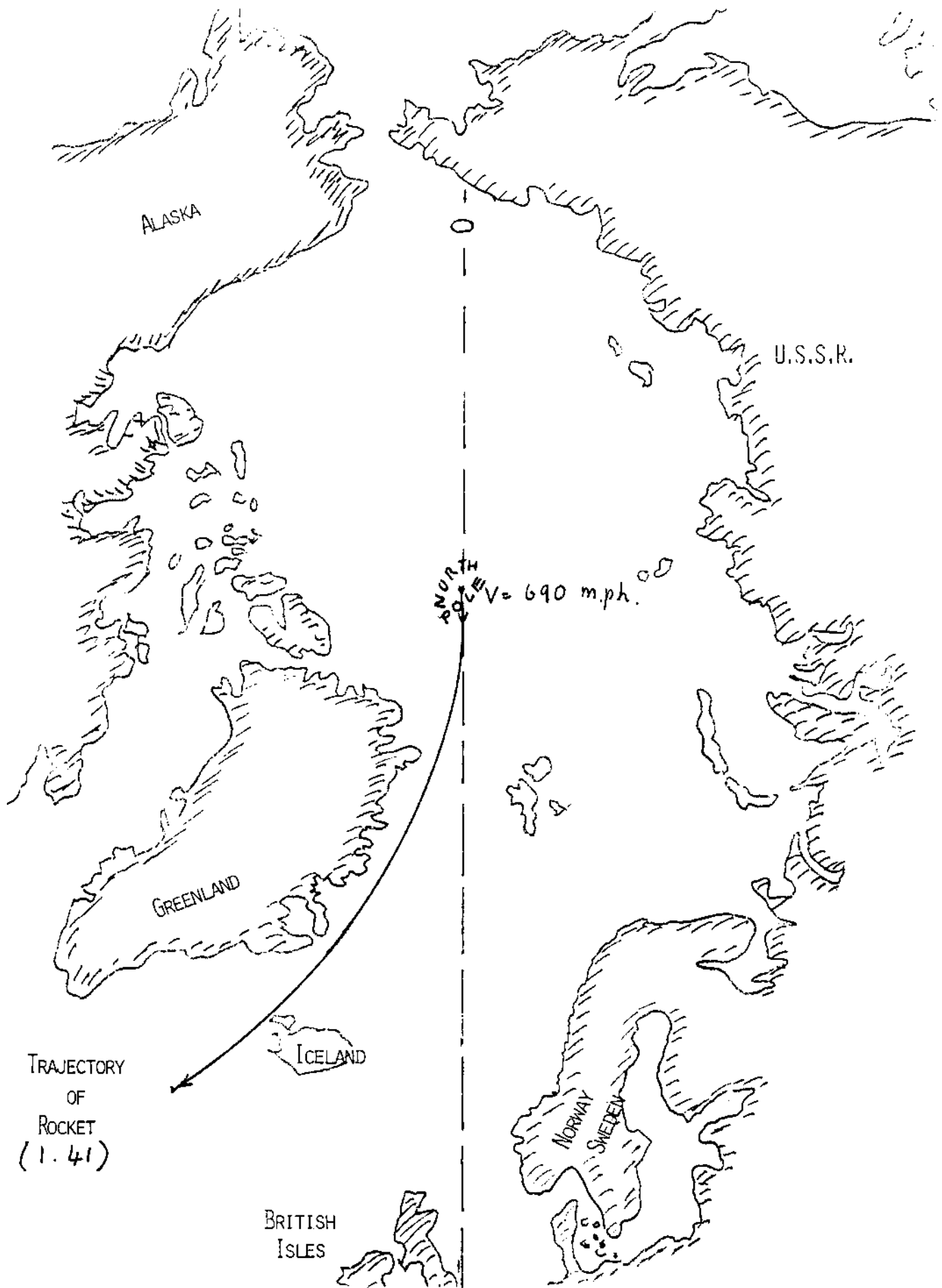


Set the pendulum swinging and rotate the bottom of the tetrahedron.

Qualitative Question: Is the swinging of the pendulum affected by rotating the base?

The equivalent pendulum on earth would be at the north or south poles. Would the swing of the pendulum rotate with the earth? To demonstrate this, place the pendulum over the center of the polar map of the world on the next page. The fictitious force which appears to rotate the phase of the pendulum is called the Coriolis force.





Experiment: 1.41 The Coriolis Effect

Materials: String, book, tape, marble, paper, cup.

Method: Trying to explain why an object which you might think should travel in a straight line appears to move in a curved path on earth's surface is frequently a real problem. Here is about the simplest way you can demonstrate it--and showing how it occurs really helps. Tape a fairly heavy book closed and suspend it level, as shown in figure 1, by four strings fastened underneath which tie together to one string attached to a support such as a table top. Tape a sheet of plain paper to the top of the book. Dip a marble in a cup of water, start the book spinning, and roll the marble onto the paper on the book. The water on the marble will leave a quite obvious track on the paper. Replacing the plain paper with a copy of the map of the north pole, given in the Foucault's pendulum experiment is a help when referring specifically to the earth. Try rolling the marble more or less rapidly, and spin the book fast and slow. It is also interesting simply to drop the marble on the paper, and see how it travels outward. The experiment is qualitative--the general mathematics of rotating systems is quite involved and probably is not worth tackling at an elementary level, except perhaps for special cases, such as a marble with uniform velocity v starting from the axis of rotation of a book spinning with angular velocity ω . Then, its position after time t in rotating polar coordinates on the paper stuck to the book is $\theta = \omega t$, $r = vt$. An example is a cannon fired from the north pole.

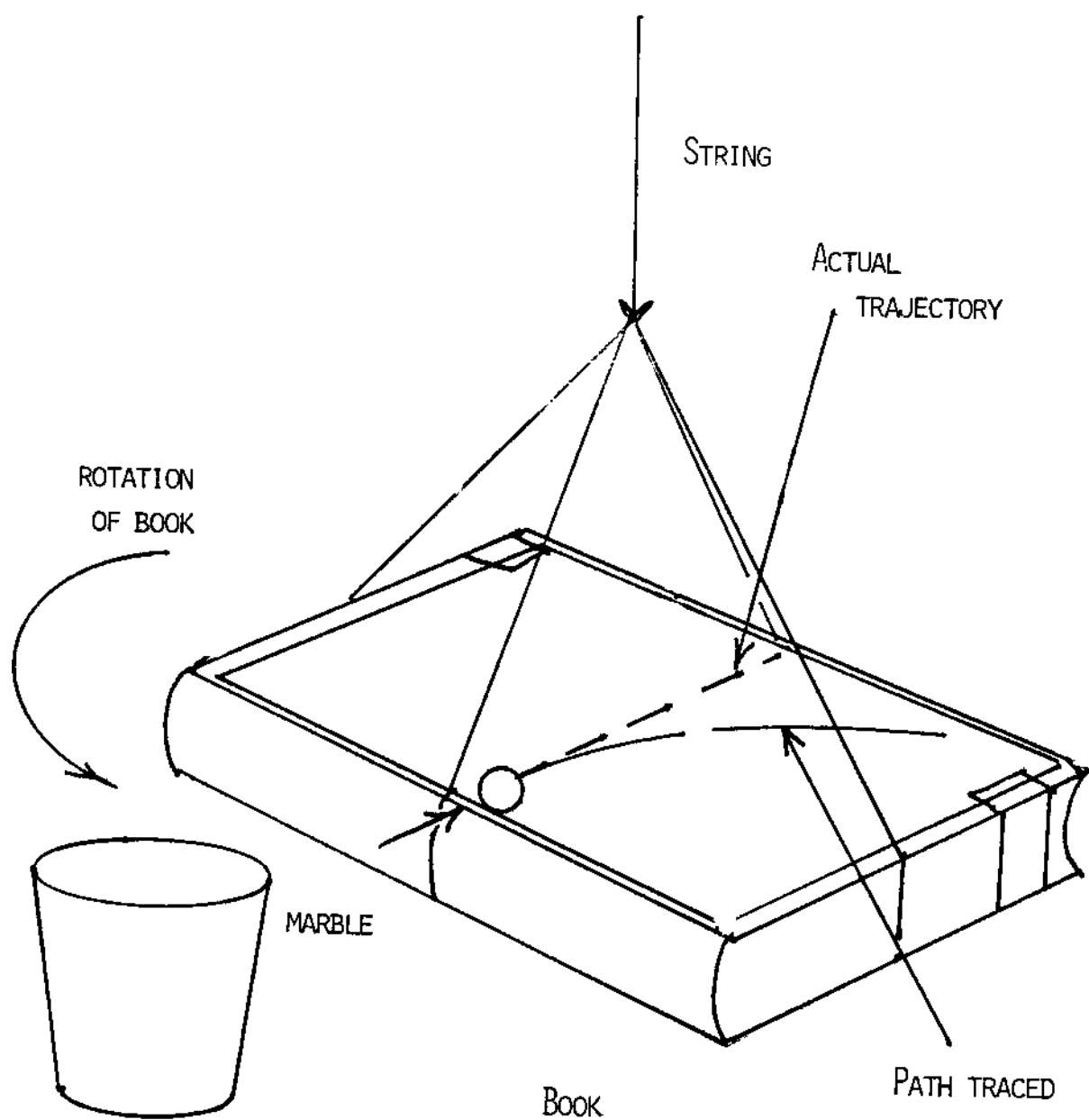
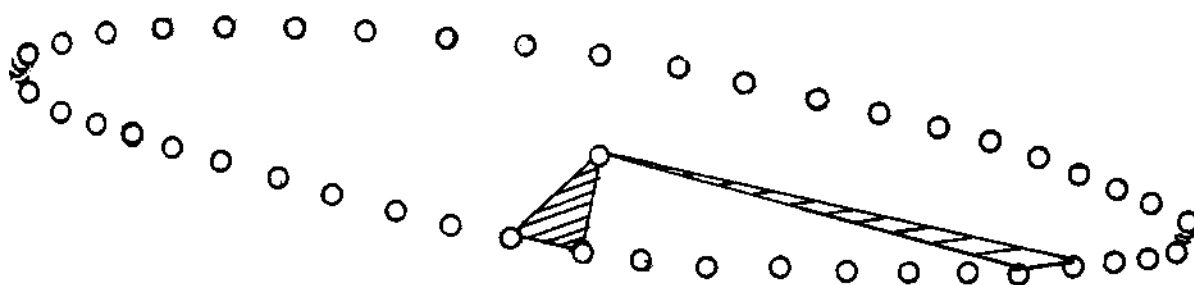


Fig. 1

Experiment 1.42 Law of Equal Areas

Materials: cups, string, straws, paper clip

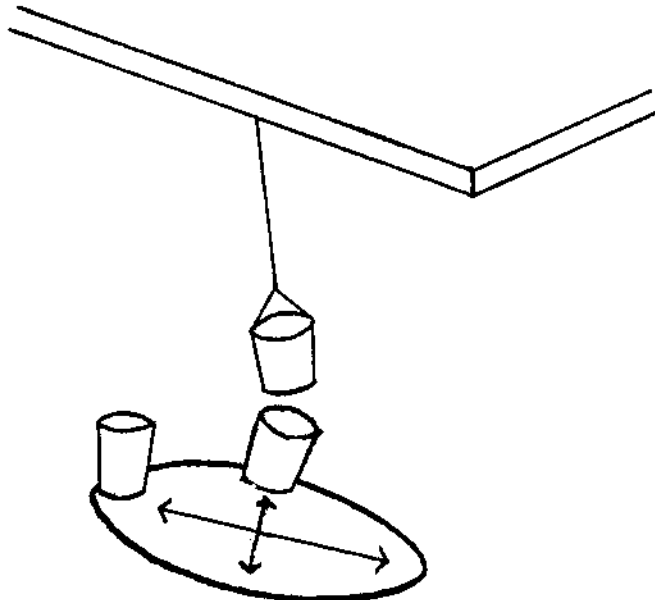
Procedure: Conservation of angular momentum under a central force leads to Kepler's law of equal areas - that a line from a planet to the sun sweeps out equal areas in equal time. This law applies to all orbits under central forces. For example, if we observe an elliptical pendulum from above illuminated by a strobe light, we get the picture shown below. The two triangles shown are equal in area.



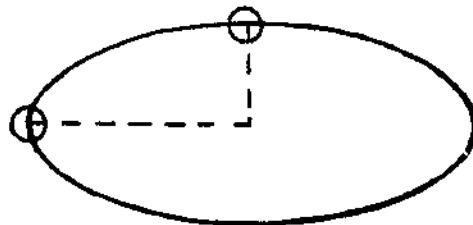
The following experiment confirms the same law in the absence of a strobe light. This is rather a messy experiment, and should be done where spilt water will do no harm. With a needle, poke a hole in the bottom of a cup. Put about fifteen marbles in the cup which is attached by a string to a suitable support, a chair back, the edge of a table, etc. Set the system going to ensure you can swing the cup in a suitable ellipse. Spread a large sheet of newspaper on the floor. Fill the cup with a diluted mixture of ink and water. The hole should allow a few drops a second to emerge. The elliptical pendulum traces its path by the ink drops falling on the newspaper, and since there are equal time intervals between the separation of the drops, the velocity of the pendulum will be proportional to the reciprocal of this distance.

Viscosity causes the amplitude of the pendulum to decay, so several orbits may be mapped.

Another method of performing the same experiment is to replace the ink in the cup with water, and enlarge the hole somewhat. After the pendulum is set swinging in an ellipse, two cups are placed under it, one at the end, and one about the middle of the swing, so that water goes into both cups as shown.



The trajectory will be mapped out by water drops on the floor, and you need to know the distance from the center, and the angle, for the two cups.

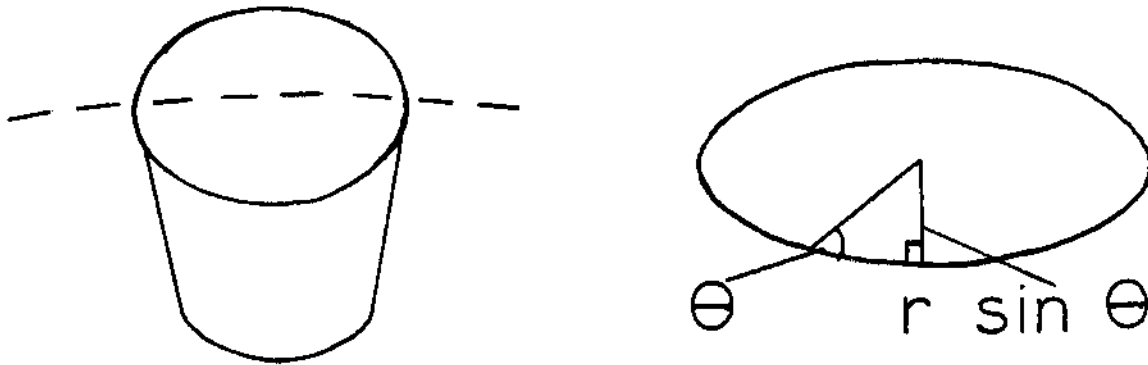


The volume, or mass, of water in each cup needs to be measured. If you let a lot run out, you can just measure the height in each cup, but then you have a real mess on the floor. If you let a little run into each cup, you can suck it up from the cup in a straw, and measure the length,

(taking care not to suck it into the mouth!) The volume measured is proportional to the time the pendulum spends over each cup, and hence the reciprocal of the velocity. The area of the triangle representing the angular momentum is therefore, proportional to $\frac{\text{moment to the center } (r \sin \theta)}{\text{mass of water in the cup}}$

Note: the trajectory must cross the center of the cup, or less will go in.

Note that the central force in this experiment is elastic- that is,



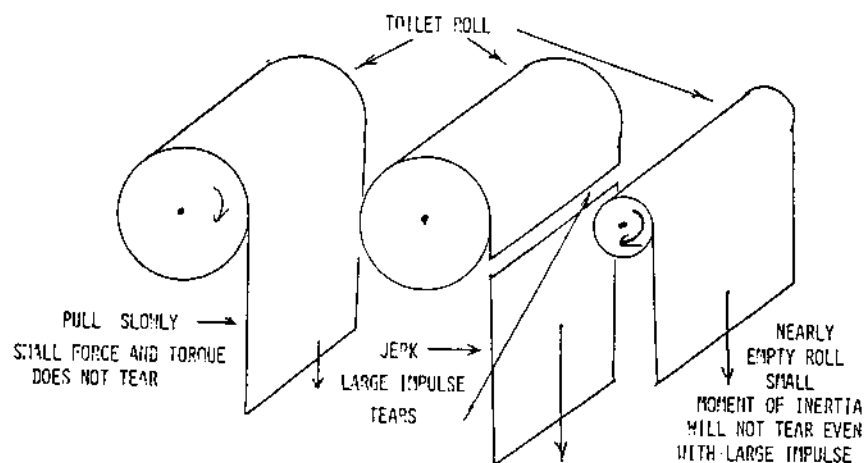
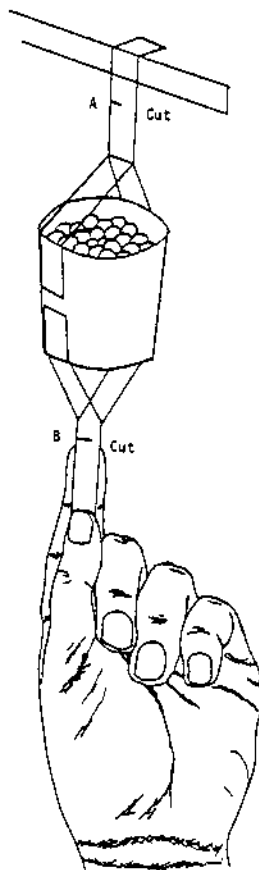
the force is proportional to the displacement, and directed towards the center. This gives an elliptic orbit with the force directed to a point at the center of the ellipse. With an inverse square law, such as provides planetary orbits, the force is directed towards the focus of the ellipse, which is close to one end for an eccentric orbit. Since angular momentum is conserved in both these cases, the law of equal areas applies. For planetary motion, the sun is the focus.

Experiment 1.43 Inertia

Materials: Tape, scissors, marbles, cup.

Procedure: Hang a cup, full of marbles, from the table, as shown, using sticky tape. Now cut two thirds the way through the tape at A and B making sure the cuts are identical. Give a very sharp pull at B. Which tape breaks? Repeat, pulling very slowly. Does the same tape break? Why not?

You make a similar experiment using rotational inertia every day. When you pull the toilet roll slowly, it unrolls without tearing, because the force on the paper, and the torque on the roll is small. Then you jerk the paper and it tears - the moment of inertia of the roll opposes the sudden torque on it, and the increased tension in the paper tears it. Why is this much more difficult when the roll is almost empty?



Experiment 1.44 Car accelerometer

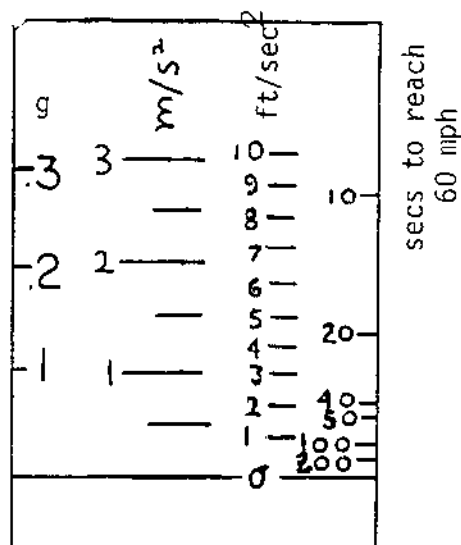
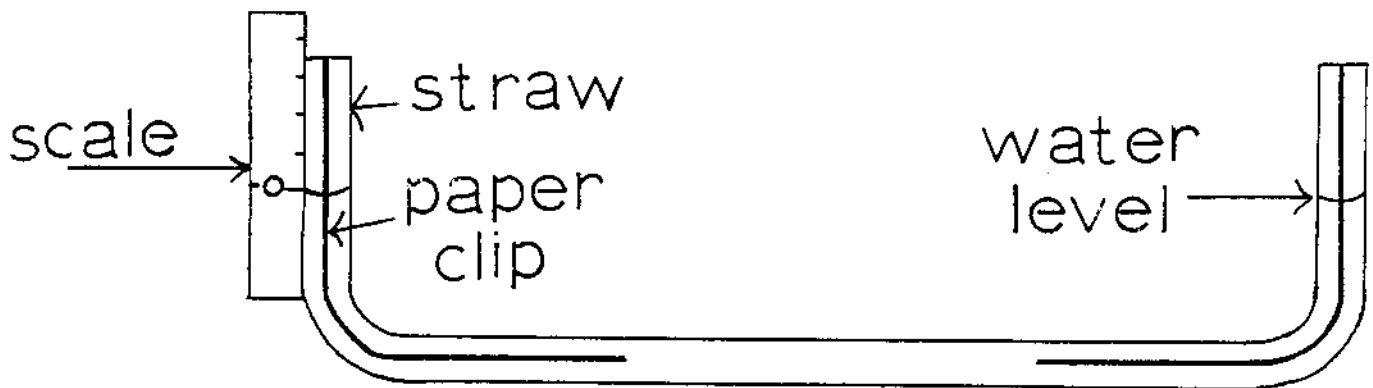
Materials: two paper clips, soda straw.

Procedure: Straighten out the paper clips and insert them in the ends of the straw. Then bend the straw to the shape shown below. Tape the device to the side window of a car, filling it with water level with the zero. Note how high the water rises when accelerating along a level road. The device can be used on a bicycle, but is very difficult to read without falling off.

If the acceleration of the car tilts the water to angle θ ,

$$\tan \theta = \frac{\text{car acceleration}}{\text{acceleration due to gravity}}$$

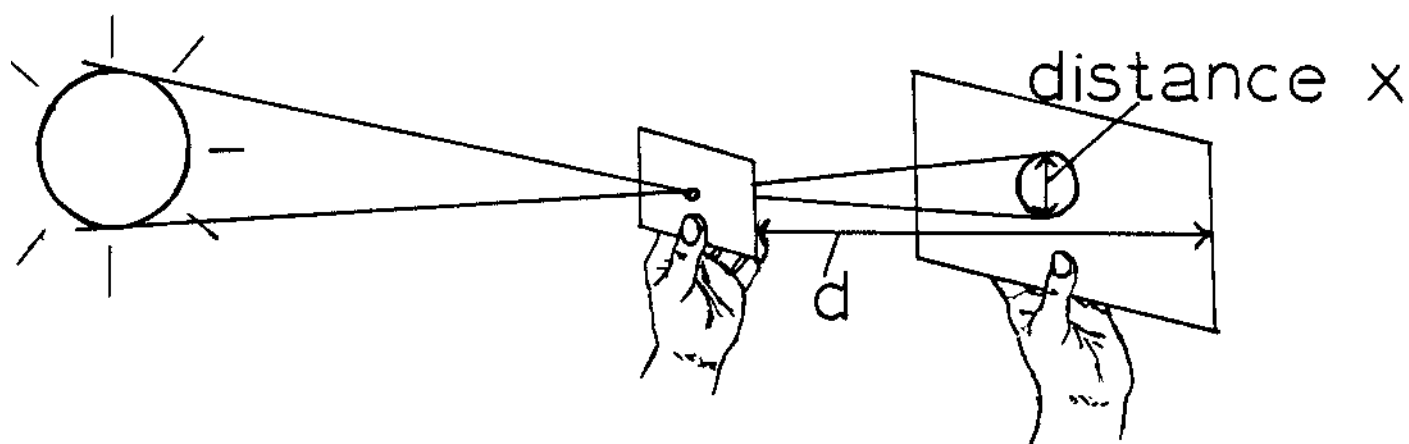
What is the maximum acceleration of the car in gs (acceleration due to gravity)? How long would it take to reach 100 km./h (63 mph) from rest with this acceleration?



Experiment 1.45 The size of the Sun

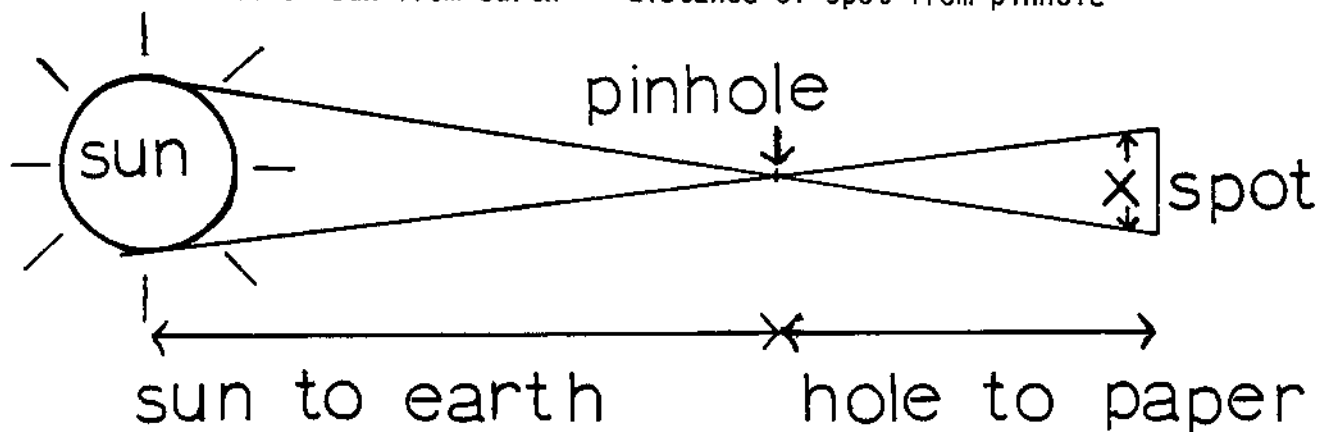
Materials: Paper, card, pencil.

Procedure: Measuring the size of objects is always interesting, from the size of the nucleus (see experiment 8.02) to the size of the universe (no experiment included). The size of the sun, and of a tall nearby building present interesting experiments. Poke a large pinhole in a sheet of card or thick paper on a sunny day, and allow the sun's image to fall on a sheet of paper a fixed distance from the hole. Make pencil marks at the edge of the

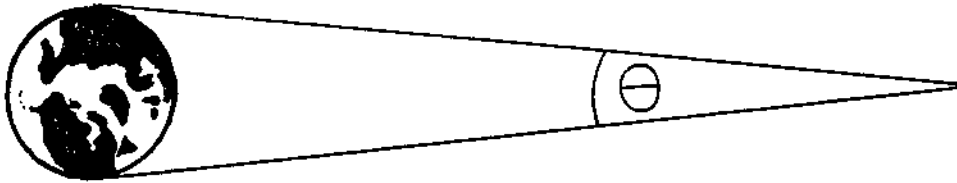


image, and measure the separation with a ruler to be X cm. Measure the distance of the pinhole from the paper to be d cm. If you look at the figure, you will see

$$\frac{\text{diameter of sun}}{\text{distance of sun from earth}} = \frac{\text{diameter of spot } X}{\text{distance of spot from pinhole}}$$



All we need to know here is the distance of the earth from the sun. This is a little more difficult, and has been obtained by using the earth as a base line by taking sights of one point on the sun simultaneously from opposite sides of the earth, to get the angle θ as shown.

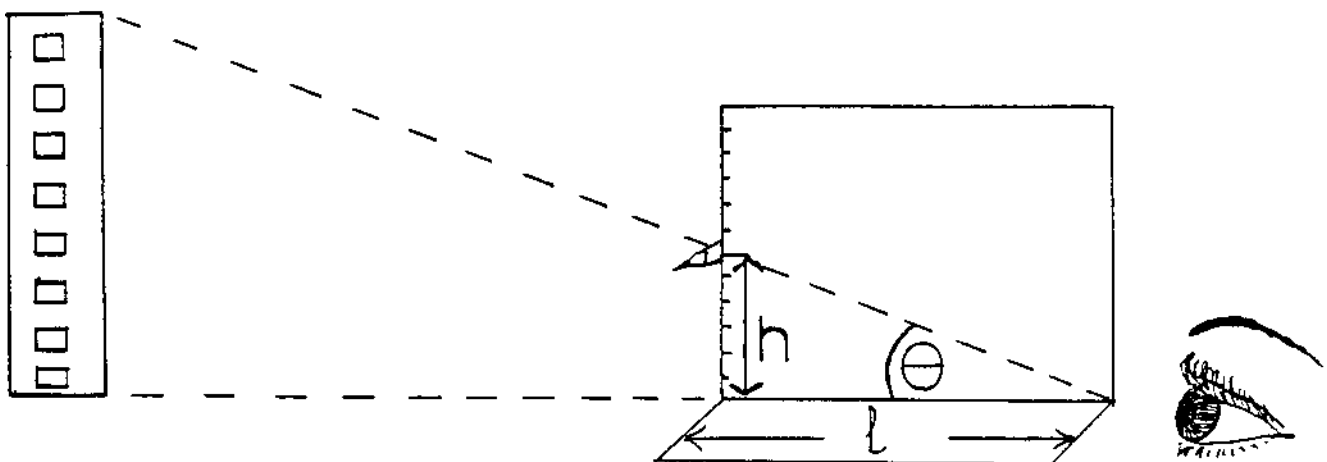


The distance to sun = $\frac{\text{diameter of earth}}{\theta}$

This gives the distance to the sun as 92.9 million miles, or 169 million Km. What is its diameter?

Now for a simple method for the heights of buildings. Pace out a suitable distance from the building, about equal to what you think is its height. Count the number of paces.

Cut out the figure on the following page, and fold it up, as shown. Aim the lower part at the base of the building, and hold a pencil in line with the top.



The edge of the sheet is graduated to give you the tangent of the angle

$$\theta = \frac{h}{l}.$$

The height of the building is then tangent of θ x number of paces x length of pace.

Measure ten paces to obtain the length of your pace. A pace is about two and a half feet, three quarters of a meter.

Length of your pace =

Number of paces =

Tan θ =

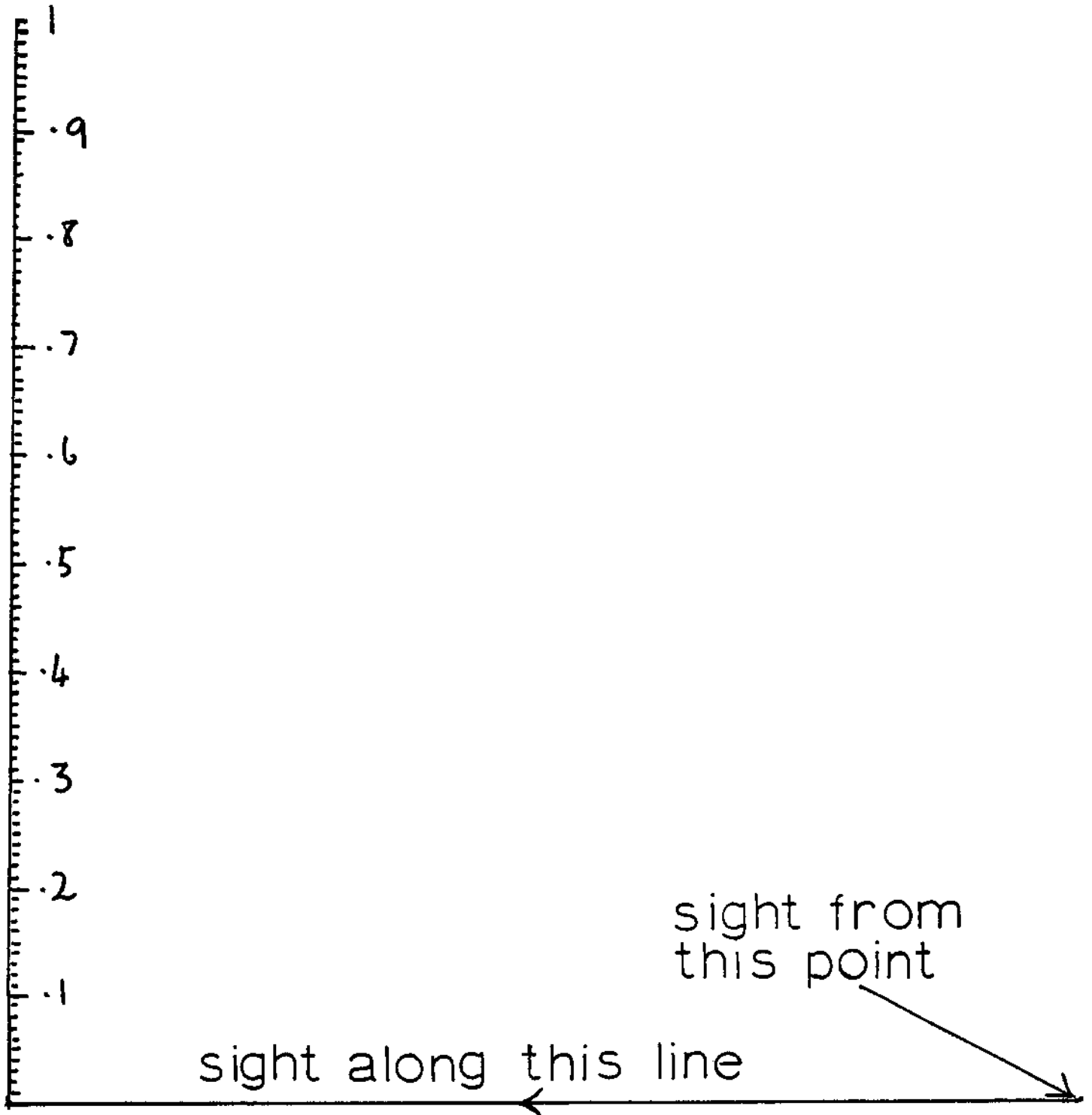
Height of building =

Repeat the whole process to get the building height.

How close together are your values?

How accurate do you think this experiment is?

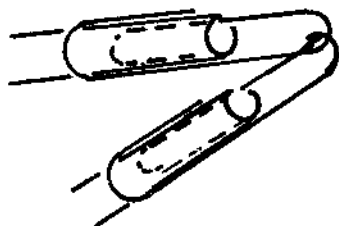
How best could it be improved?



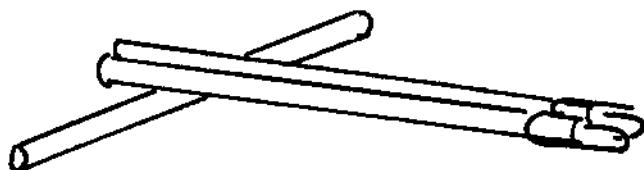
Experiment 1.46 Rolling Uphill or Antigravity

Materials: Three straws, paper clips, marble.

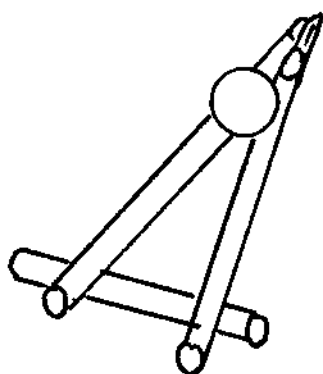
Procedure: Fasten two straws together at one end using paper clips as shown.



Place the straws together over a third straw or pencil.



Now, balance a marble at the lower end and gradually separate the straws. the marble will ultimately roll toward the upper end. Why does the marble appear to roll uphill? By moving the straws apart and together it is possible to make the marble roll completely to the top end. Can you explain this? The solution lies in the fact that by separating the straws, the marble rolls horizontally between them, the kinetic energy being provided by the center of mass falling slightly. Before the marble drops to the table the straws are brought rapidly together. This increases the potential energy, but does not appreciably affect the kinetic energy so the marble continues to roll to the upper end.

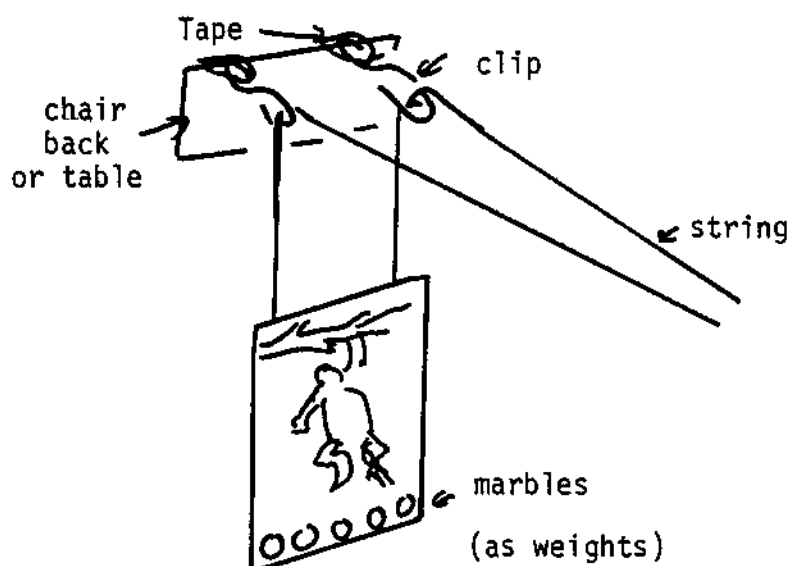


Experiment 1.47 The Monkey Puzzle

Materials: Marbles, rubber band, paper clips, string, tape.

Procedure: A famous problem in physics is the monkey (or squirrel) problem. A hunter aims a gun (or bow and arrow) so that the gun (or arrow) points directly at a monkey (or squirrel) hanging from a tree. The question is, as the hunter fires, should the monkey hang from his branch, or let go?

To see what the monkey should do, let us perform the experiment. Cut, or tear, out the monkey overpage, and attach five marbles along the bottom of the page, to weight it. Attach two strings from the top of the picture, pass the strings through two paper clips attached to the back of a chair, or table, as shown below.



To make the gun, roll a sheet of paper around a pencil and tape it as shown in the figure. Tape a rubber band to the paper cylinder, as shown, and tape the cylinder to a book, as a support. Sight down the tube to aim at the monkey, about three or four feet away, which can be raised or lowered by its supporting strings. Stretch the rubber band over the end of the pencil in the tube, and hold pencil in the taut position, together with the strings supporting the monkey, with one finger. Release the strings and pencil simultaneously.

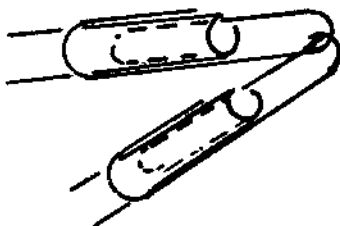
When does the pencil strike the monkey?

Should the monkey hold on or let go when the hunter fires? Why?

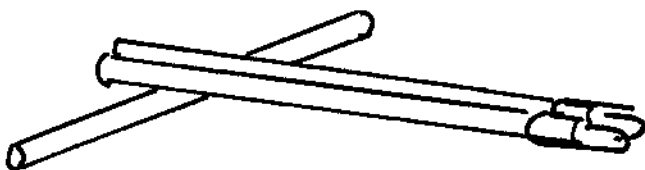
Experiment 1.46 Rolling Uphill or Antigravity

Materials: Three straws, paper clips, marble.

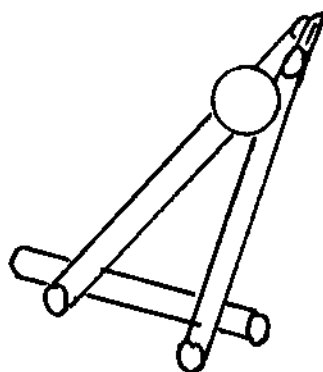
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Now, balance a marble at the lower end and gradually separate the straws. the marble will ultimately roll toward the upper end. Why does the marble appear to roll uphill? By moving the straws apart and together it is possible to make the marble roll completely to the top end. Can you explain this? The solution lies in the fact that by separating the straws, the marble rolls horizontally between them, the kinetic energy being provided by the center of mass falling slightly. Before the marble drops to the table the straws are brought rapidly together. This increases the potential energy, but does not appreciably affect the kinetic energy so the marble continues to roll to the upper end.

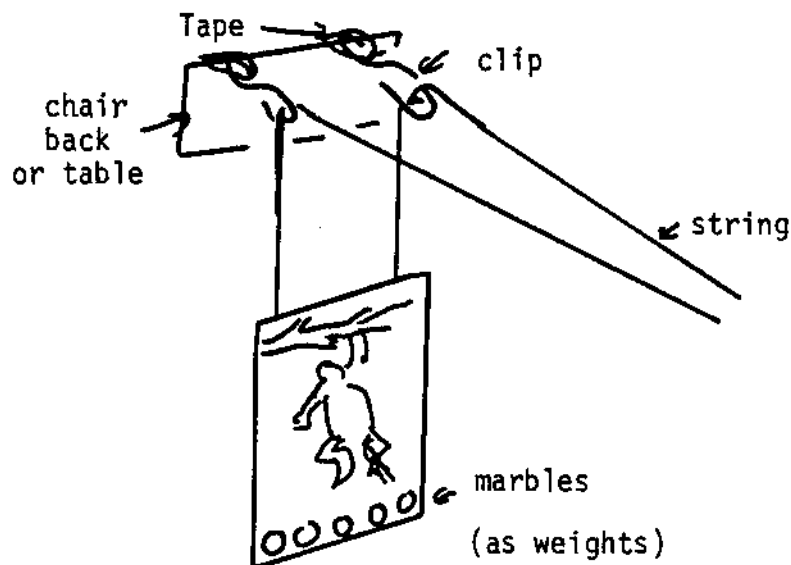


Experiment 1.47 The Monkey Puzzle

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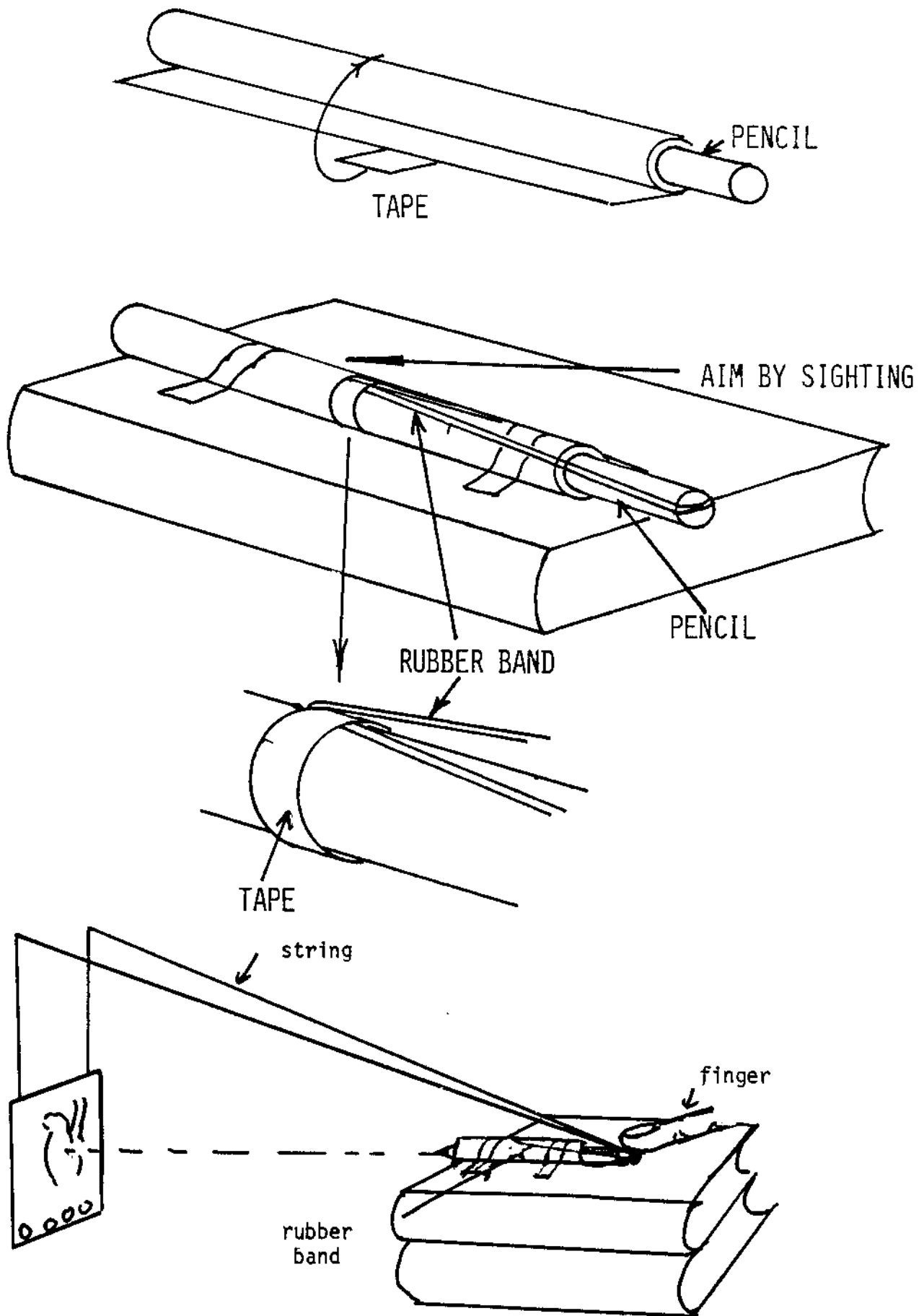
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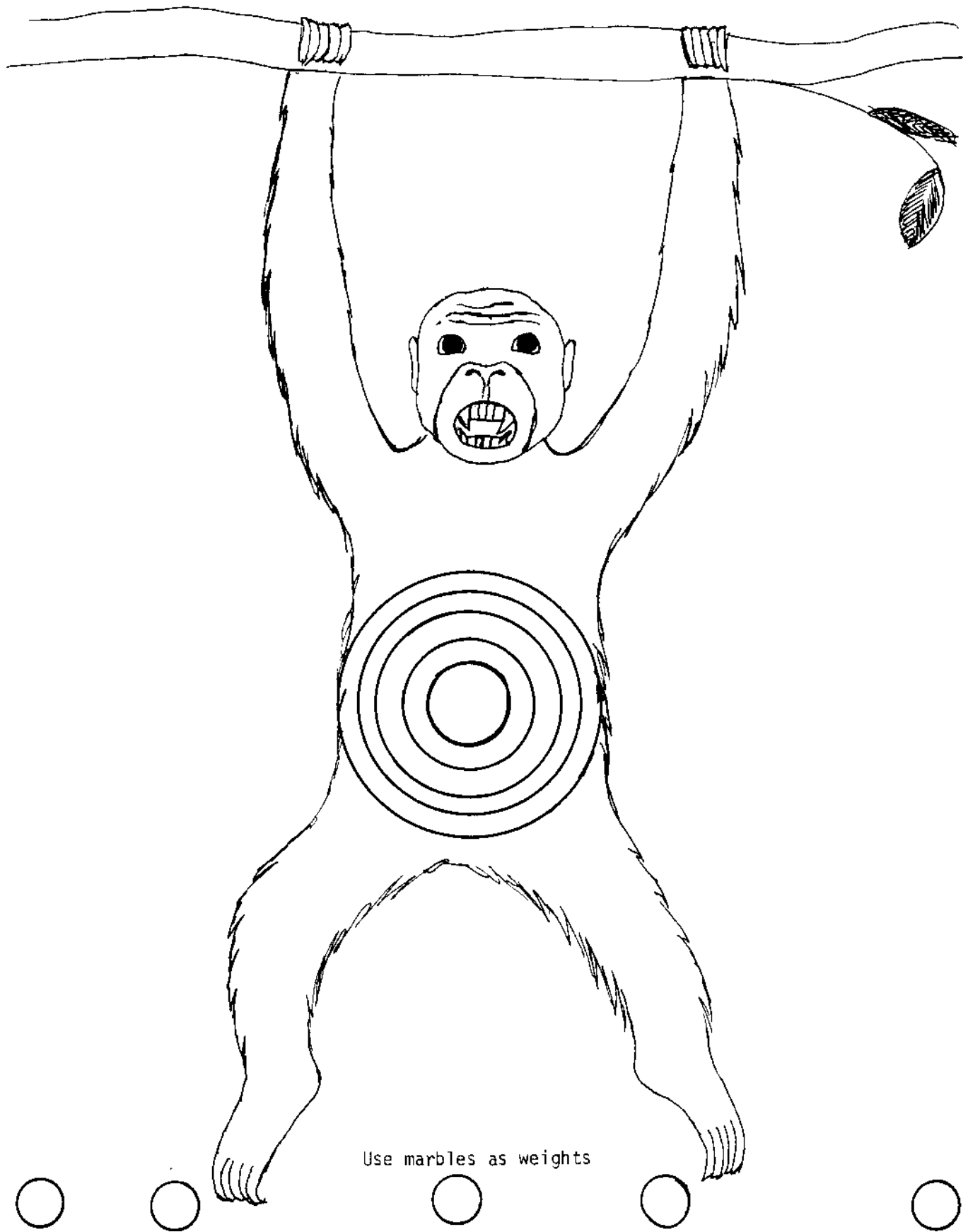


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When does the pencil strike the monkey?

Should the monkey hold on or let go when the hunter fires? Why?





Experiment 1.48 Angular momentum of coins

Materials: Two quarters and a nickel.

Procedure: Three coins are held as shown in figure 1, the nickel between the two quarters. The bottom coin is then released, holding on to the top coin. It is nearly impossible to release both sides of the quarter at the same instant, and the side first released starts at once to turn about the opposite side. This turning motion continues as the coin falls, and the nickel falls below the quarter, which is very odd to the uninformed observer. If it takes a fall of about ten inches for the coins to rotate 180° , many students believe it will take twenty inches for the coins to land with the nickel again above the quarter. In fact, of course, it is 40 inches. Once released, the coins rotate at a constant speed and so form a clock, since there is no torque, and the distance fallen will be proportional to the square of the angle of rotation. If we assume the quarter is 2.5 cm in diameter the velocity of the center must be about 15 cm/s at the point when the edge of the coin ceases to stick to the finger, to provide the correct angular velocity. Kinematically, this occurs after the center has fallen little more than a millimeter. In mentioning this to John Webb at Sussex University, he said "Oh yes - in fact you can win quite a lot of money in pubs with that experiment."

If, instead of simply dropping the coins you give them a slight twist as they are released, as shown in fig. 2, they do not turn over at all! The secret is in the gyroscopic forces involved. The twist makes the coins spin about a vertical axis. The small torque given the coins as they fall from the fingers, which previously turned them over, now merely gives a slight precession about the vertical axis.

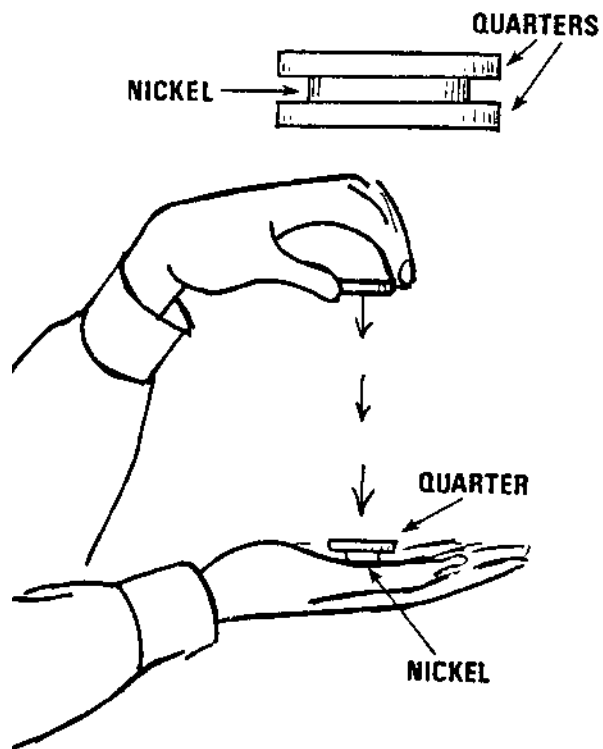


Fig 1.

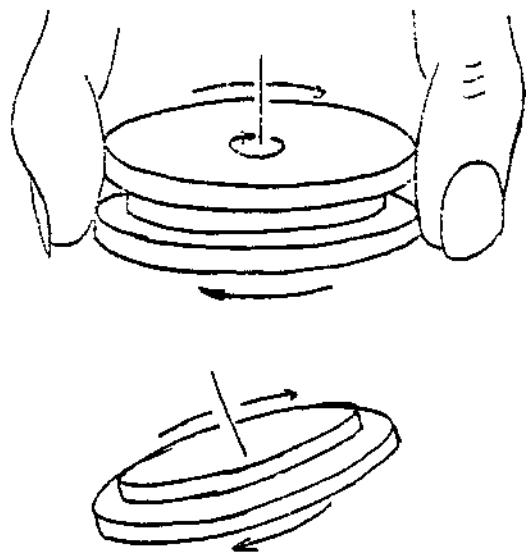


Fig 2.

EXPERIMENT 1.49 Mobiles and Moments

Materials: marbles, straws, string

Method: The usual examples given for illustrating moments tend to be rather dull--balances of various kinds or levers lifting barrels. Mobiles provide an interesting, elegant and artistic way of demonstrating the principles of moments. The simplest way is merely to use combinations of marbles, attached to straws suspended by fine threads or strings. Paper clips can be used in place of marbles.

One example is shown in figure 1.

Moments give us that

$$\begin{array}{lcl} a \times 3 = b \times 1 & & \text{(Three marbles hung from a, one from b)} \\ c \times 1 = d \times 1 \\ e \times 2 = f \times 4 \\ h \times 6 = g \times 12 \end{array}$$

Do experiments agree with theory?

Instead of marbles, we can use abstract cardboard cutouts, or other objects. The mass of the cutouts can be obtained from their area, for which one can count squares if they are drawn on squared paper--so the mass of the two objects drawn in figure 2 would be as 140 to 275 or $L \times 140 = M \times 276$. The mobile can be informative in other ways--one can make an atom, with a nucleus such as He^4 , as shown in figure 3, or the planetary system (figure 4) -- though only the home planets can be displayed to scale, the others would be too far!

Furthermore, the abstract mobile shapes provide interesting experiments in the center of mass as they are supported from different positions.

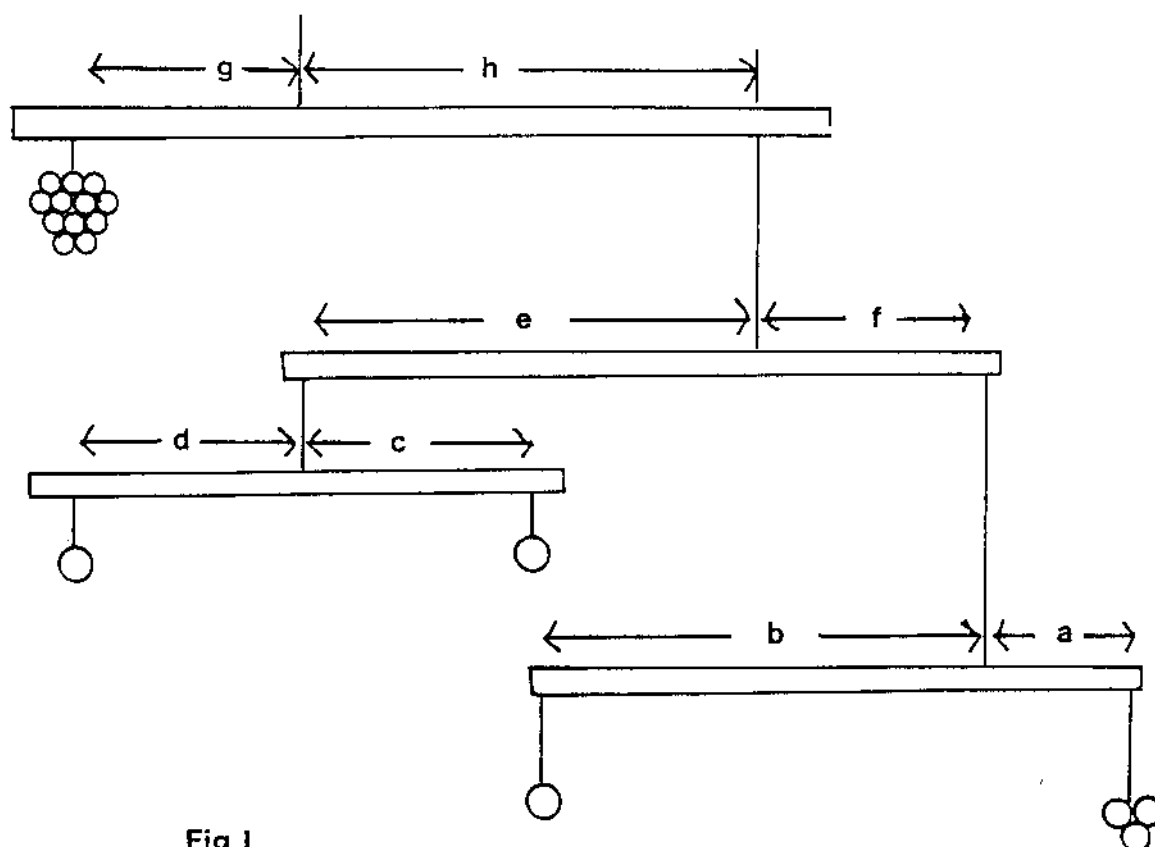


Fig 1

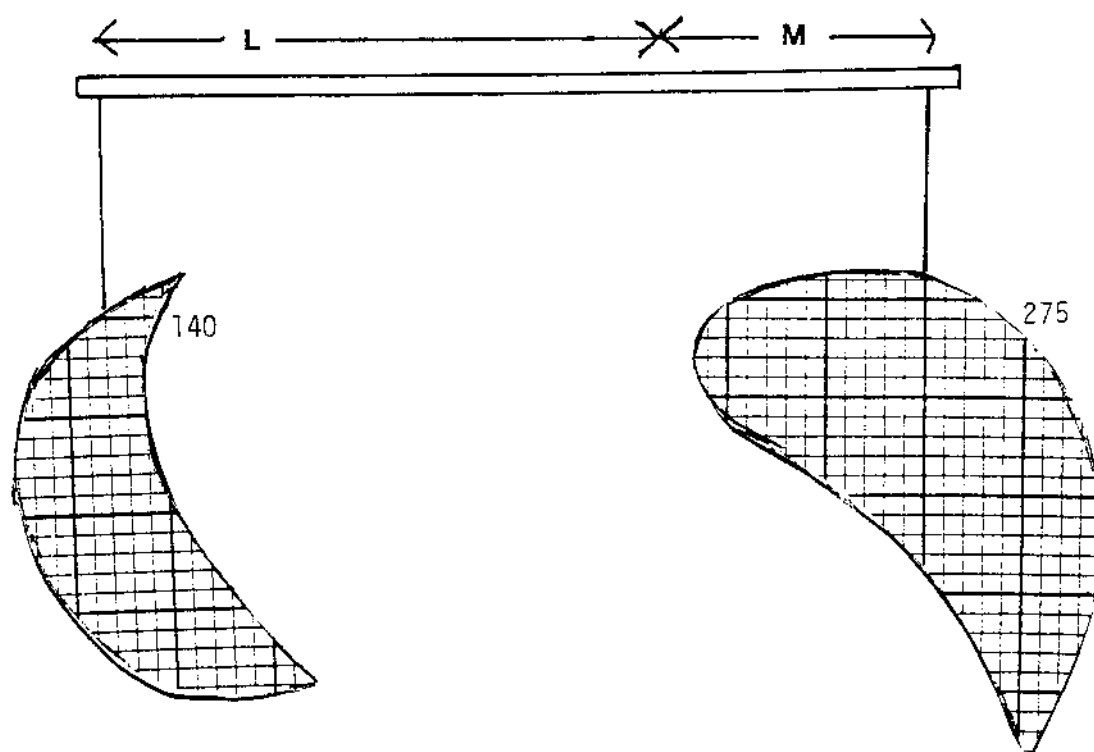


Fig 2

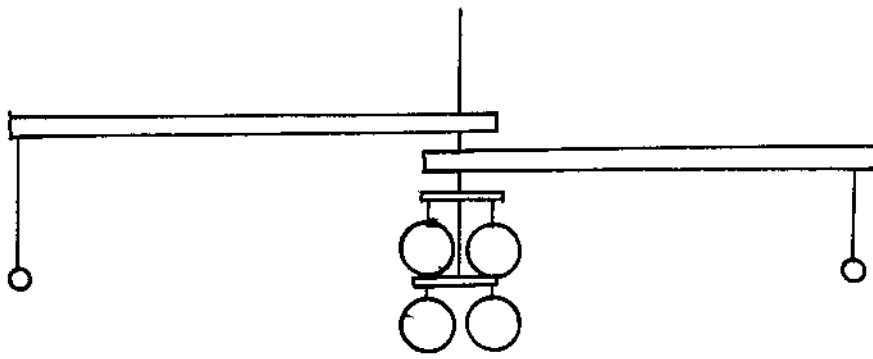


Fig 3

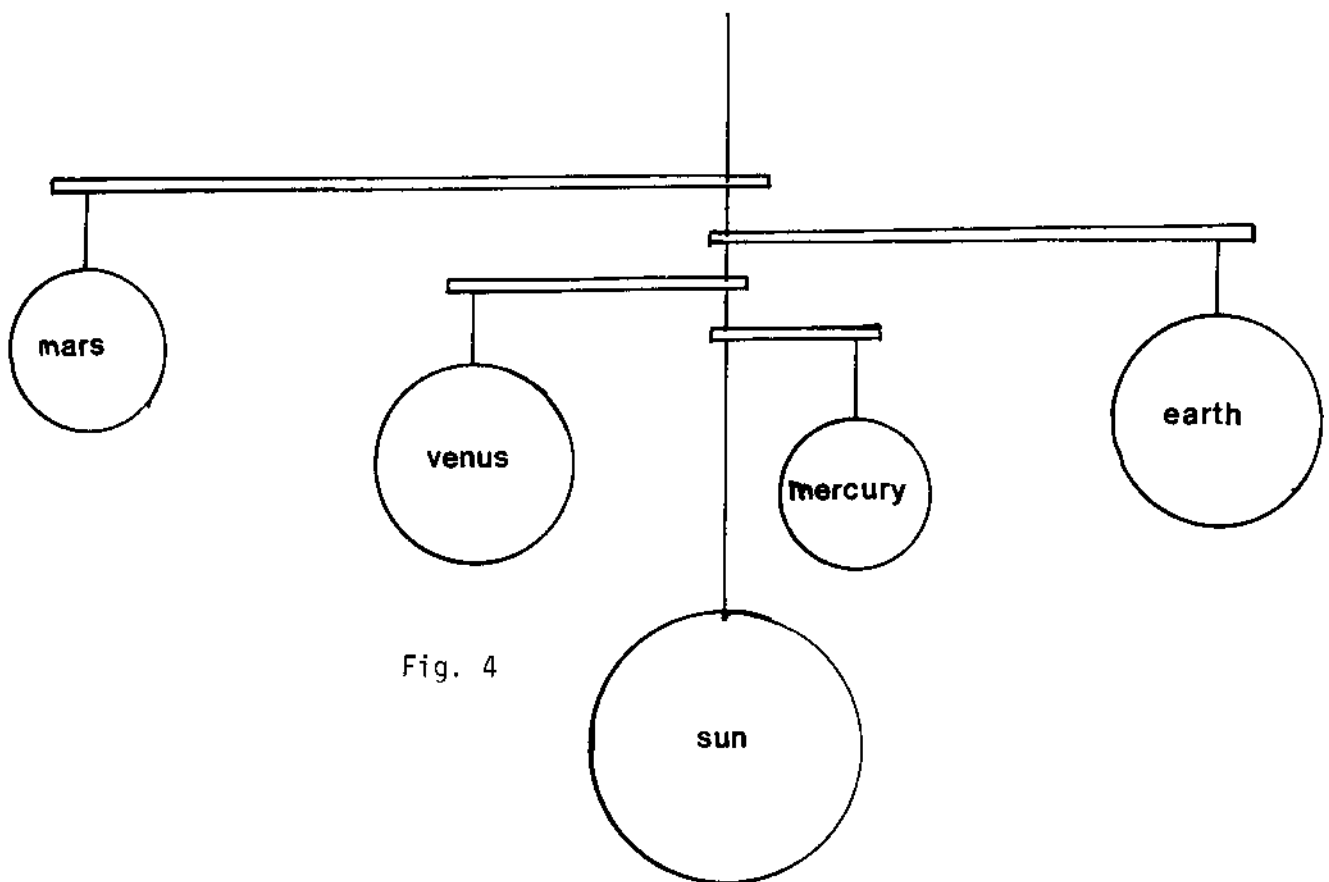
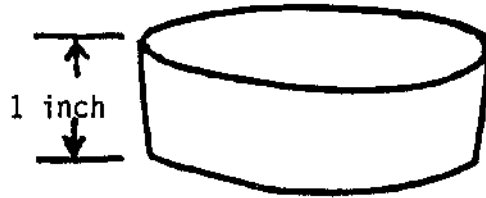


Fig. 4

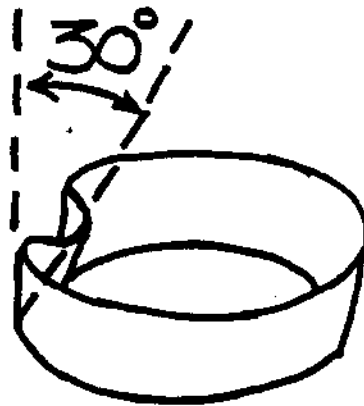
EXPERIMENT 1.50 Angular Acceleration - the ball and ruler

Materials: ruler, tape, cup, marble

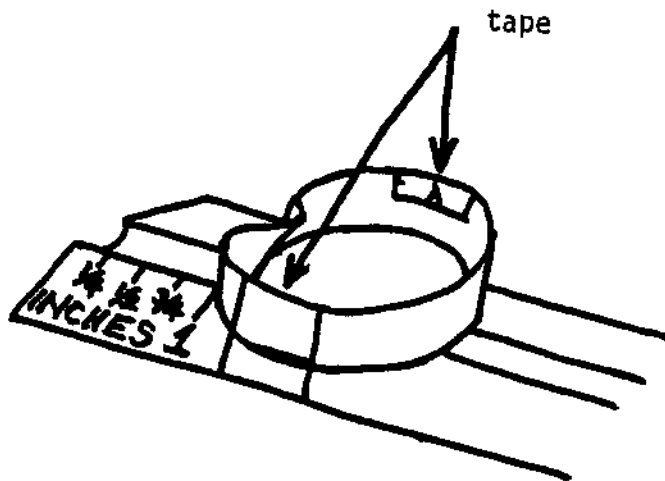
Method: Cut about 1" off the bottom of a styrofoam cup.



Push one side in so that it sits at about 30° to the vertical.

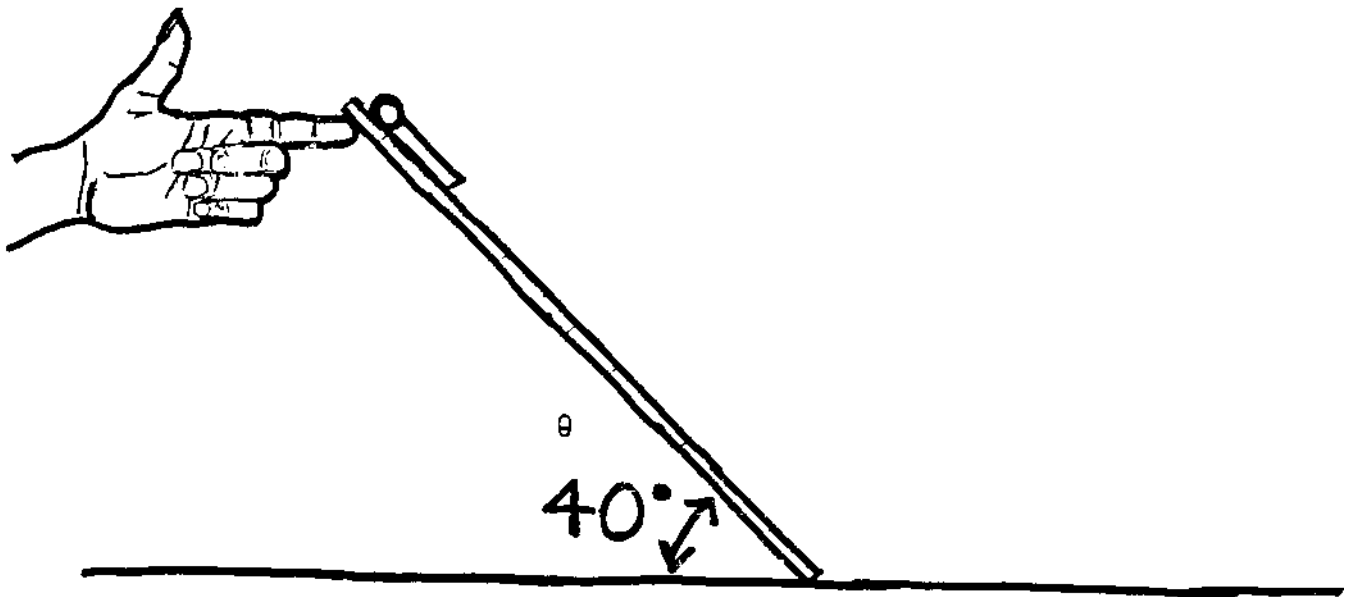


Now tape the cup one inch from the top of the ruler.



Hold the ruler at about 40° to the horizontal, with one end on the floor, as shown. Place a marble against the cup. Release the ruler, and the marble jumps into the cup. The device works best on a carpet which cushions the collision with the floor, which can make the marble jump out of the cup again. Why does the marble jump into the cup?

On a smooth table, it may be necessary to tape the end of the ruler to the table, to act as a hinge, and prevent slipping.



The cup accelerates more rapidly than g , and hence drops away from the marble. Roughly, we may say the center of the ruler wants to accelerate at g , and for this to be true, the end must accelerate more rapidly. Let us show how this works mathematically.

If the angle θ is varied, then only below an angle θ_c does the stick drop away from the ball.

We can see this from the equation of motion of the ruler, where I is the moment of inertia of the ruler about one end since the rate of change of angular momentum $I\dot{\theta}$ equals the moment of the applied force. I is the moment of inertia of the ruler, and M its mass. Also, from geometry,

$$I\ddot{\theta} = -Mg \frac{L}{2} \cos \theta \quad (\ddot{\theta} = d^2 \theta / dt^2, \dot{\theta} = d\theta / dt)$$

$$I = \frac{1}{3} ML^2$$

$$\text{Hence, } \ddot{\theta} = \frac{3g}{2L} \cos \theta$$

The end of the stick has a linear acceleration of

$$\ddot{S} = L \ddot{\theta} = -\frac{3g}{2} \cos \theta,$$

whose vertical component is

$$\ddot{Y} = \ddot{S} \cos \theta = -\frac{3g}{2} \cos^2 \theta$$

Now, this only exceeds the acceleration due to gravity if $-\frac{3}{2}g \cos^2 \theta < -g$
or $\cos^2 \theta > \frac{2}{3}$

which is true for angles smaller than about 35° . Miller, in his book "Physics Fun and Demonstrations," points out that this effect is related to the way in which tall chimneys buckle as they fall, since there is a torque trying to accelerate the end of the chimney faster than g when the angle is below 35° , which will break the chimney.

EXPERIMENT 1.51. Inertia, Momentum, and Blow Football

Materials: straws, paper, marble

Procedure: Most people have played blow football--a small light ball is placed between two contestants who endeavor, by blowing through tubes, to propel the ball over the opponents edge of the table. This note is to suggest the physical principles behind the game. Play one game using a marble and two straws. Now place a dampened piece of paper between the palms, and roll it into a ball about the same size as the marble. Play the game again and ponder the following questions:

In which game was it easier to get the ball moving?

The light paper ball is more mobile, and can easily be turned around--we say it has less inertial mass than the marble. The marble nicely exemplifies Newton's first law--a body continues in its state of rest or uniform motion in the absence of forces. The force you can deliver by blowing is the same in both cases, but you have to blow much longer to stop the marble, because for the same force, if the mass is higher, the acceleration (or deceleration) is slower, according to Newton's third law.



Experiment 1.52 Quantum Properties
(Suggested by Fream B. Minton)

It is often difficult to convey the idea of the quantum nature of the physical properties of substances. As an exercise in determining such properties, different numbers of marbles are placed in cups which are sealed with paper on top, so the number is not visible. Only marbles obtained from one source have a sufficiently constant mass to be used, and this should be checked. The number of marbles in each cup is arranged so that the differences between cups are both odd and even numbers of marbles, and no two cups are separated by a single marble. One empty cup is supplied, and the smallest number of marbles in a cup is five. The object is to determine the mass of one "quantum". Table I shows how the cups were prepared.

In the second part of this experiment the students are given a brief description of the famous experiment by Robert Millikan in which he determined the charge of a single electron. The students are not told his analytical method, only how data was obtained and what the numerical values mean. Pointing out that Millikan's problem was very similar to the "balls in the cup" problem, the instructor supplies a sample of data in which oil-drop charges are recorded. The students are asked to use this information to find the charge on a single electron. Table II is a sample of the kind of information supplied to the students. Although somewhat contrived, it serves the purpose.

By the time this part is reached, many of the students have discovered for themselves the principle that quantum units may be measured by determining differences between measured values and looking for the largest common factor. As a result, most of them are quickly successful in finding the charge on a single electron.

As a thought question, the instructor might ask if Millikan was justified in throwing out a small amount of data he obtained which indicated that the charge on an electron was one-third of the value he ultimately reported.

TABLE I

<u>Can No.</u>	<u>No. of Balls</u>
1	0
2	5
3	11
4	13
5	16
6	20
7	27

TABLE II

<u>Oil Drop Event</u>	<u>Electric Charge</u> <u>10^{-19}^x Coul.</u>
1	6.4
2	16.0
3	20.8
4	28.8
5	32.0
6	46.6
7	67.2
8	94.4

EXPERIMENT 1.53

The Brachistochrone - or, the longer way round may be the quickest way home

Students often find it difficult to distinguish between acceleration and velocity. If an object were not accelerated, the shortest path between two points in terms of both time and distance, would be straight line. The next simplest case occurs when a constant force acts on the body. Students find it hard to believe that the shortest time to travel between the two points does not now occur along a straight line unless the force acts along the line between the two points. The brachistochrone problem - to find the path producing the shortest travel time between two points for an object under constant gravitational force - exemplifies this difficulty. It was solved by John Bernoulli (1667-1748), and led to the formal foundation of the calculus of variations. In the 18th century, the Bernoulli family occupied a large number of mathematics chairs in the universities of Europe. John himself was an irascible individual - being "violent, abrasive, jealous, and when necessary, dishonest ... His son Daniel (who developed "Bernoulli's principle" in hydrodynamics) had the temerity to win a French Academy of Sciences prize which his father had sought. John gave him a special reward by throwing him out of the house."¹

What would be the simplest way to attack the brachistochrone problem? It should be noted that, although there is a constant vertical force, the force and hence the acceleration along the trajectory need not necessarily be constant. Experimentally, we could construct a series of curves joining two points considered. Then we could try each curve to see which path could be traveled in the shortest time. The calculus of variations performs such an iteration mathematically, varying the path to determine which provides

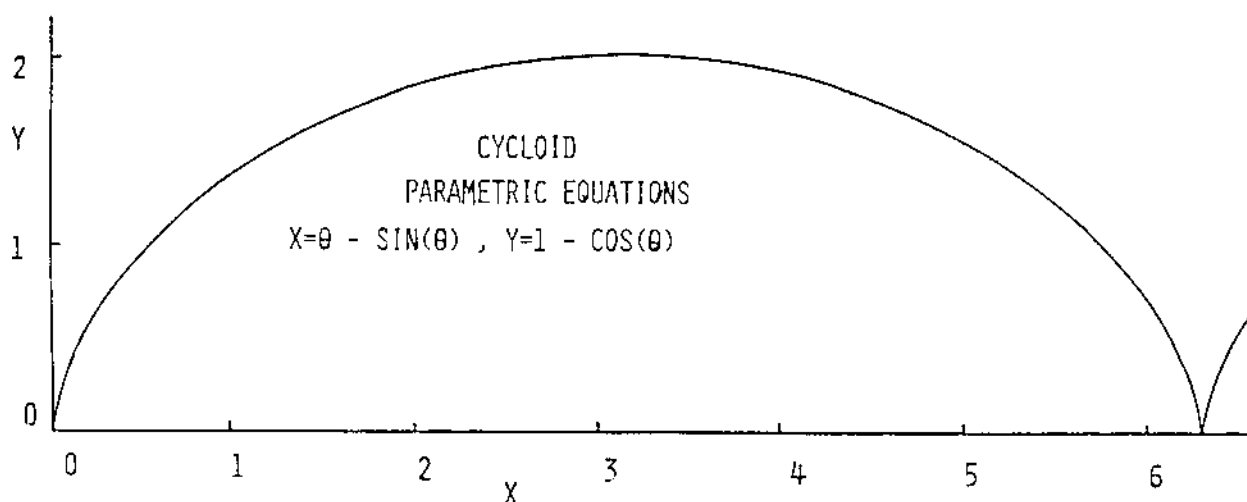
the shortest time. The solution turned out to be a cycloid - the curve traced by a point on the circumference of a wheel rolling along (you may remember the question - which part of a fast-moving train is stationary? That part of the wheel in contact with the track. The subsequent motion of the point in contact with the track forms the cusp of the cycloid).

It is easy to draw a cycloid. Tape a piece of string to the end of a can, wrap it one or two times around the can, and tape the other end to a ruler, laid on a sheet of paper. Wrapping the string forces the can to roll along the ruler without slipping. Now, hold the tip of a pencil against the can (you can dent the can to make a groove for the pencil tip to sit in) and, as you roll the can along the ruler, the pencil traces the cycloid. The size of the can will determine the dimensions of the cycloid.

For students with little physics or mathematics, the mere words "brachistochrone" and "cycloid" are offputting, however, it is possible to devise a simple experiment to show that the cycloid (or a curve close to it) provides the path of shortest time. Common sense arguments can be used to reinforce this result. Since the time taken to travel between two points is the distance divided by the average velocity, it would be a big help to accelerate rapidly at first, and quickly gain a high velocity. The cycloid starts off almost vertically (Fig. 1), so we get the maximum acceleration at first. This means it takes a shorter time for the path than a straight line incline, which provides constant acceleration. However, for a curve having a more extended vertical portion, the increased distance from start to finish is not compensated by the larger vertical acceleration: Our experiment provides three paths, a straight line, a cycloid, and a path with a larger initial vertical drop.

The cycloid is outlined in a thick black line in Fig. 2. The figure may need to be enlarged - we found an $8\frac{1}{2} \times 11$ sheet of paper or card works well, but anything larger is better. The simplest arrangement is to use sticky tape to stick straws over the section indicated in the figure. We approximate the curves by a series of straight lines, which works well for the purposes of timing the ball along the trajectory, and it is easy to enlarge the figure - relative distances are marked to help with this. It would be an improvement to tape a piece of narrow rubber hose, if available, along the cycloid curve.

Place the sheet on a large book or piece of stiff cardboard and tilt it, so that the bottom of the sheet remains horizontal. Start two marbles rolling along the cycloid and the straight line simultaneously. That rolling along the straight line clearly takes longer than that on the cycloid since it will arrive at the bottom behind the other. Now start one on the cycloid and one on the bottom path - again the cycloid wins. One can vary the speeds by tilting the book more or less. The principle is the same, with the curves vertical, although it is not possible to demonstrate it with this little experiment.



Reference

1. J. R. Newman, Vol. II World of Mathematics (Simon and Schuster, New York, 1956).

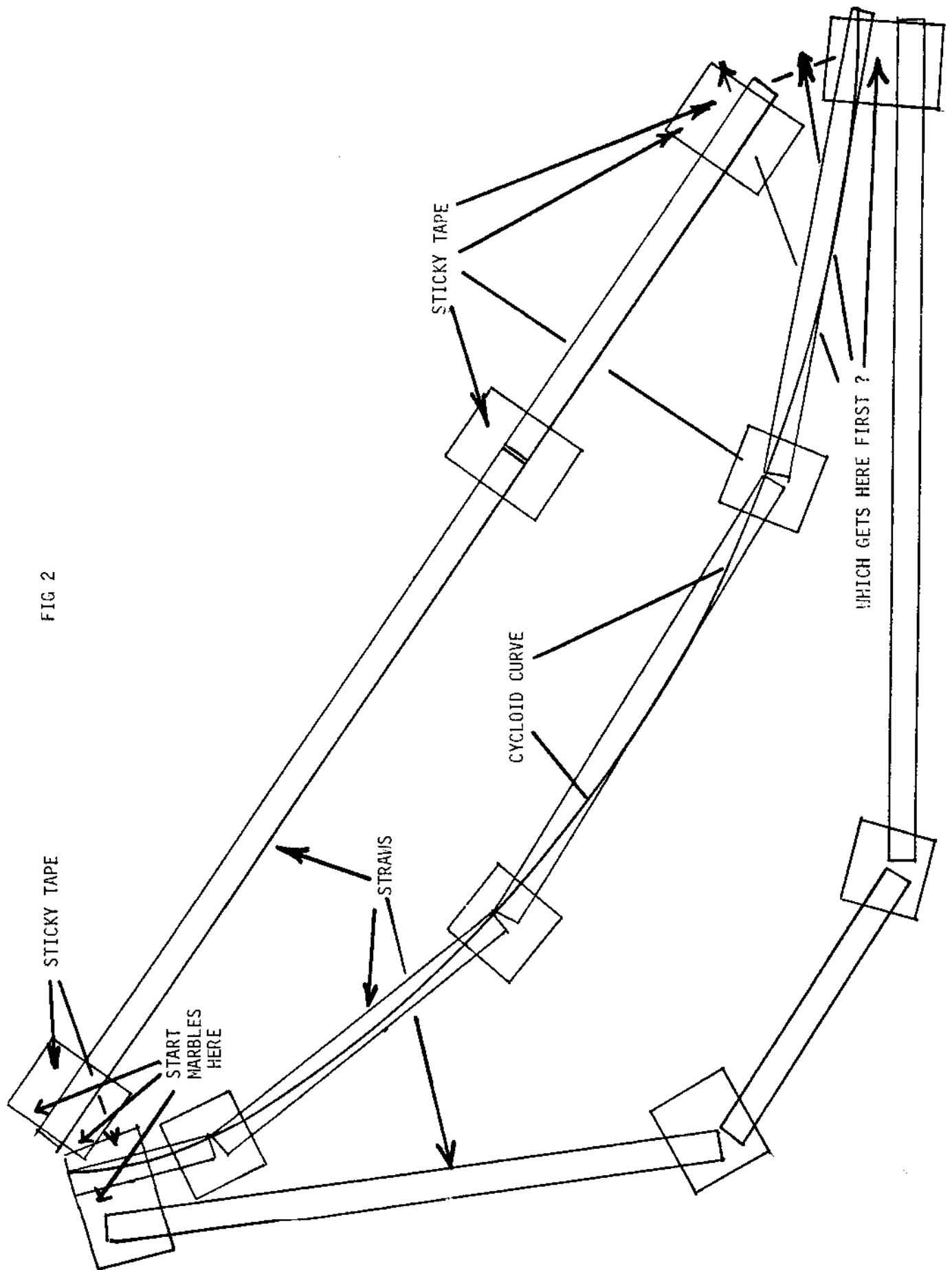


FIG 2

Experiment 1.54

Levitating a Marble - Motion in a Circle

If you cut the bottom off a styrofoam cup, can you use that cup to pick up a marble without touching the marble with your fingers? Place the cup over the marble, and swirl it round and round, as shown in figure 1. When the rotation speed is fast enough, the marble will rise up the wall, because of the reaction of the cup on the marble. The taper of the cup provides a vertical force which balances the gravitational attraction. The cup can then be safely lifted without the marble falling out. We can even measure the constant of gravitation g from this. Let the radius of the cup at which the marble rotates be r , the angle of the cup wall be θ° to the vertical and the number of rotations per second be n Hz. The period, T is then $1/n$ sec. The velocity, v , of the marble is $2\pi r/T$, and the radial force is $mv^2/r = m(2\pi r/t)^2/r$. Since the reaction of the cup is normal to its surface, the ratio of the vertical force, mg , to the horizontal radial force must be

$$\tan \theta = mg/mr(2\pi/T)^2 = g/r(2\pi/T)^2$$

Using a 6 oz. styrofoam cup, the internal diameter of the top is 7.4 cm, and of the bottom 4.8 cm. The cup (less the bottom) is 8.6 cm high, so

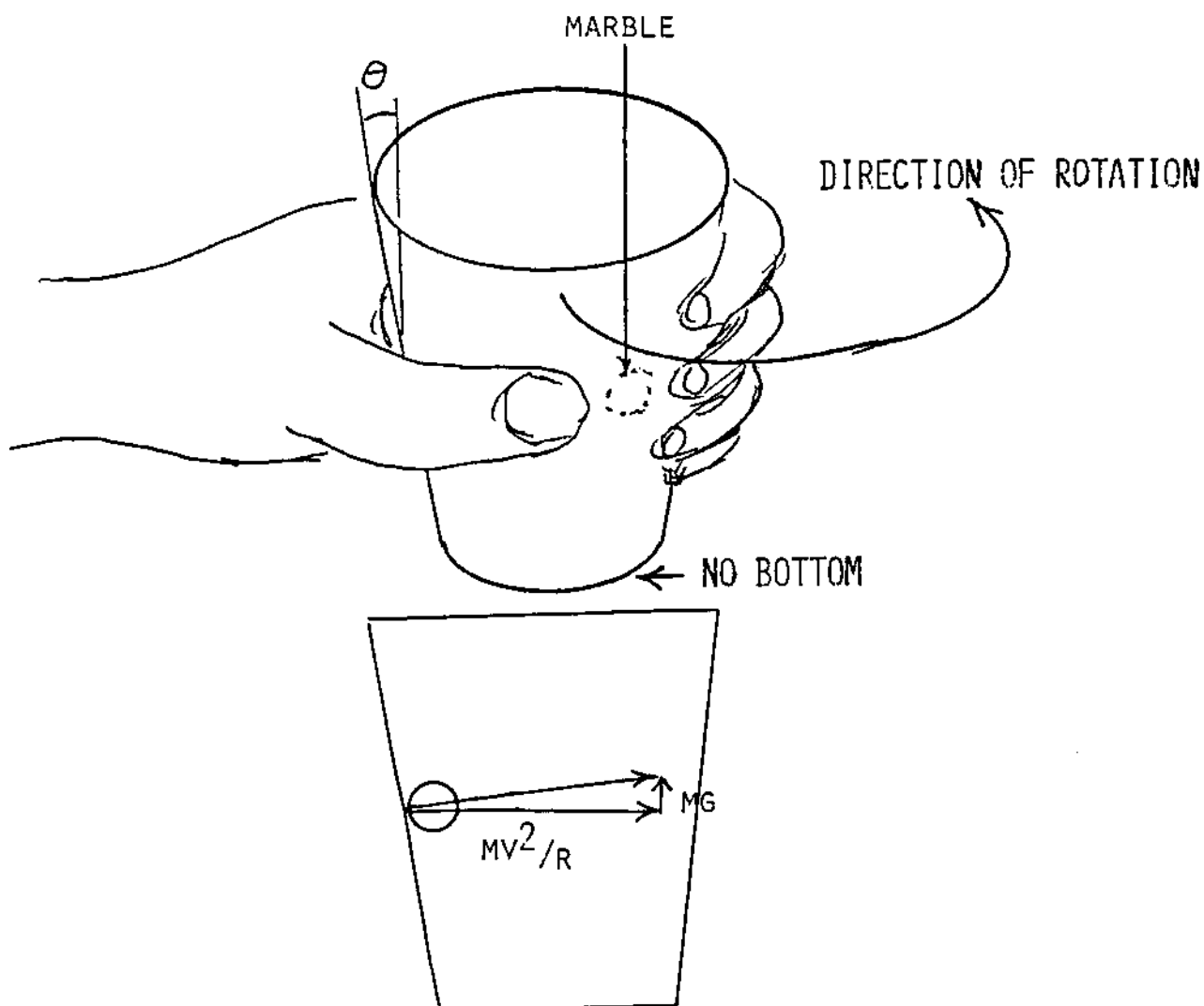
$$\tan \theta = (7.4 - 4.8)/2 \times 8.6 = 0.15$$

Now, from the previous formula

$$T = 2\pi\sqrt{r \tan \theta / g} \quad (1)$$

If we take it half way up the cup, r is 3.05 cm, from which we subtract the radius of the marble 0.8 cm. With these values, $n = 8.54$ Hz. Experimentally, it lies between 7 and 9 Hz, because the marble moves up and down the cup. If you quit after getting the marble to spin round the top of the cup the marble loses energy to friction, descends the cup, and kinetic and gravitational potential

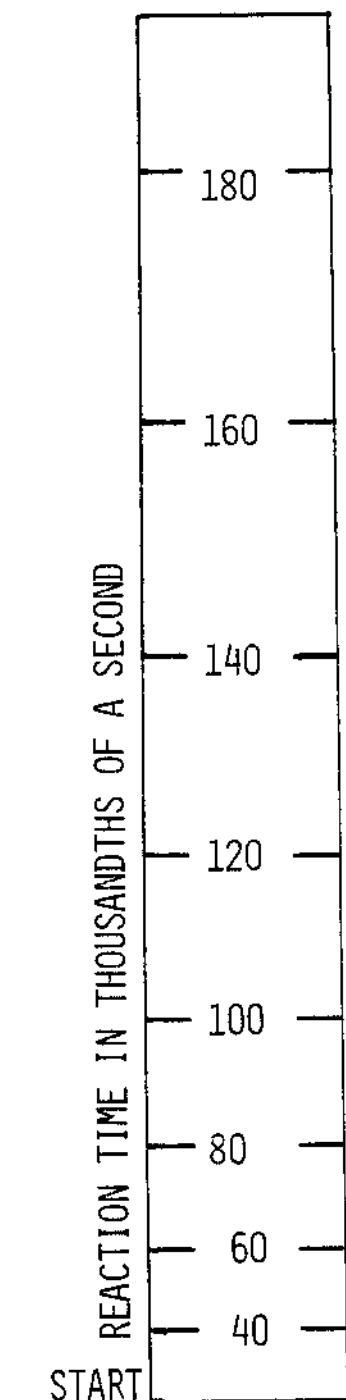
energy is converted to heat. Since the velocity is given by $v = g\sqrt{r/\tan \theta}$, and r is decreasing the marble will slow down, but equation (1) shows the period decreases with r , so it will make more revolutions per second. (This experiment is a product of the Minnix & Carpenter Summer Institute at V.M.I.)



EXPERIMENT 1.55

An application of g - the reflex tester

The test of someone's reflexes whereby one person hold a dollar bill vertically and the other holds thumb and fore finger on opposite sides of the bill. He (or she) is told they can have the bill if they can catch it by pinching it. In fact, it is virtually impossible to catch the note - one's reactions are too slow. Using $S = \frac{1}{2}gt^2$, it is easy to mark on a piece of paper -- as shown here, the time it takes the sheet to fall that distance. Hence, we have a calibrated value for our reaction time.



Experiment 1.56 Trajectories, Drops and the Strobe

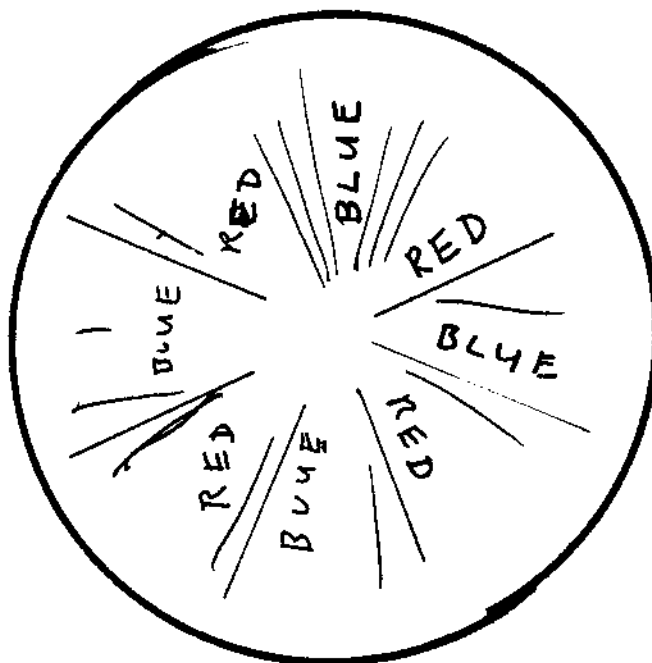
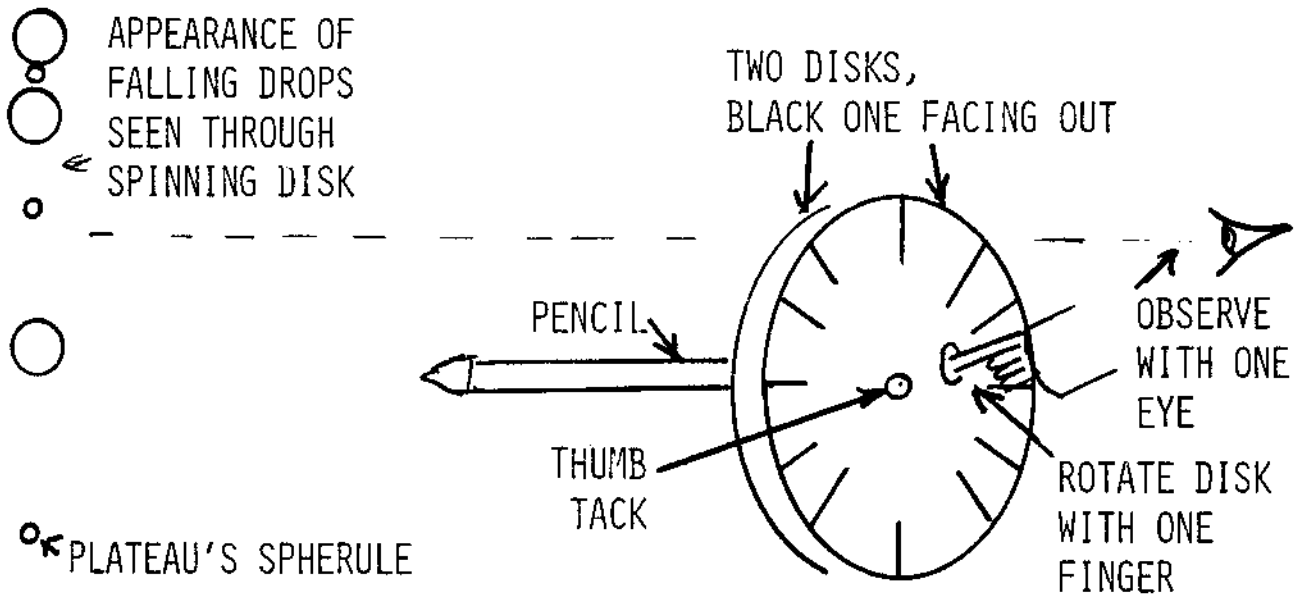
In addition to psychophysical experiments simulating motion using persistence of vision, a stroboscopic disk is a handy device for many other occasions - for example, although a fluorescent light looks white, in fact, for part of the A.C. cycle it is blue, and for part red, averaging out to white. Let's see how we can observe this.

Figure 1 shows how to construct a stroboscope with a wide range of speed. We need this because the speed with which we can spin the disk is restricted. It is not possible to spin the disk very rapidly - about 2-3 Hz is as fast as convenient - nor can one spin it too slowly - variable friction causes a variable speed. Cut out the disks in figures 2 and 3. It is best to glue them on to thick card or thin plywood and attach them to a wooden support with a pin. However, more simply, after cutting out, they can be attached back to back with four paper clips round the edge. Push a thumb tack through the center of both, with the black disk outward - then into the eraser end of a pencil, as shown. The disks can now be spun with one finger and the fluorescent light observed. At a suitable speed, a stationary pattern as shown will be seen. One can vary the number of slits on the disk from one to twelve by rotating one disk with respect to the other. The alternating stripes of blue and red occur because the mercury discharge part of the light occurs over less than half a cycle, and is bluish, the fluorescent part of the light over the full cycle, and is reddish. The spectrum is shown in figure 4.

Now, adjust a water tap so that a fine stream of water falls from it, breaking into drops just before striking the sink. You can adjust the speed of the strobe to the point where a drop appears stationary. Actually, of course, you are seeing successive drops separated by the time it takes the strobe to move the angle between two slits. Notice how the

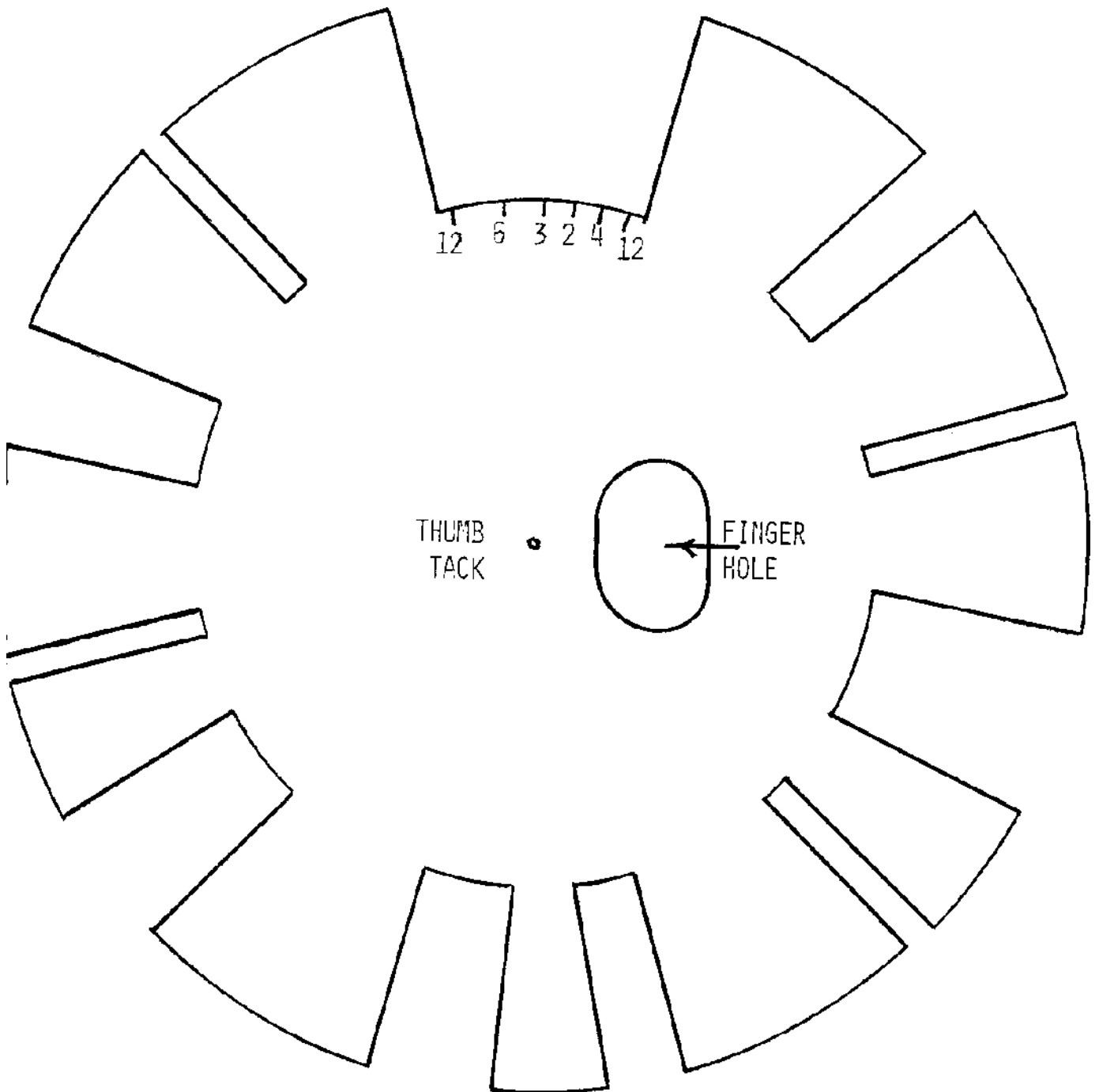
distance between the drops increases as they fall because of the acceleration due to gravity. The way in which the stream breaks up into drops is interesting - there is one tiny drop (called "Plateau's spherule") between any two big drops.

What other things can we observe through the strobe? Anything which is periodic - a tuning fork, a ruler held to the table at one end and weakened to vibrate. One interesting object acoustically is a bowed violin. A cello is easier to see, being bigger. Put the cello on the table and have someone bow it. Look along the length of the string through the strobe. One thinks of the violin string as having a uniform vibration too and fro. What you will see is a spike (called the Helmholtz spike) running up and down the string. A guitar string, when plucked, has two spikes running in opposite directions, which die out rapidly. An electric fan can be made to appear stationary - but it can also be made to look as though it was moving backwards. Which is moving a little faster, the fan or the slots in the disk, to make the fan appear to move backward, assuming they move at the same rate when the fan appears stationary?



APPEARANCE OF
FLUORESCENT LIGHT
ON LOOKING THROUGH
ROTATING STROBE

Fig 1



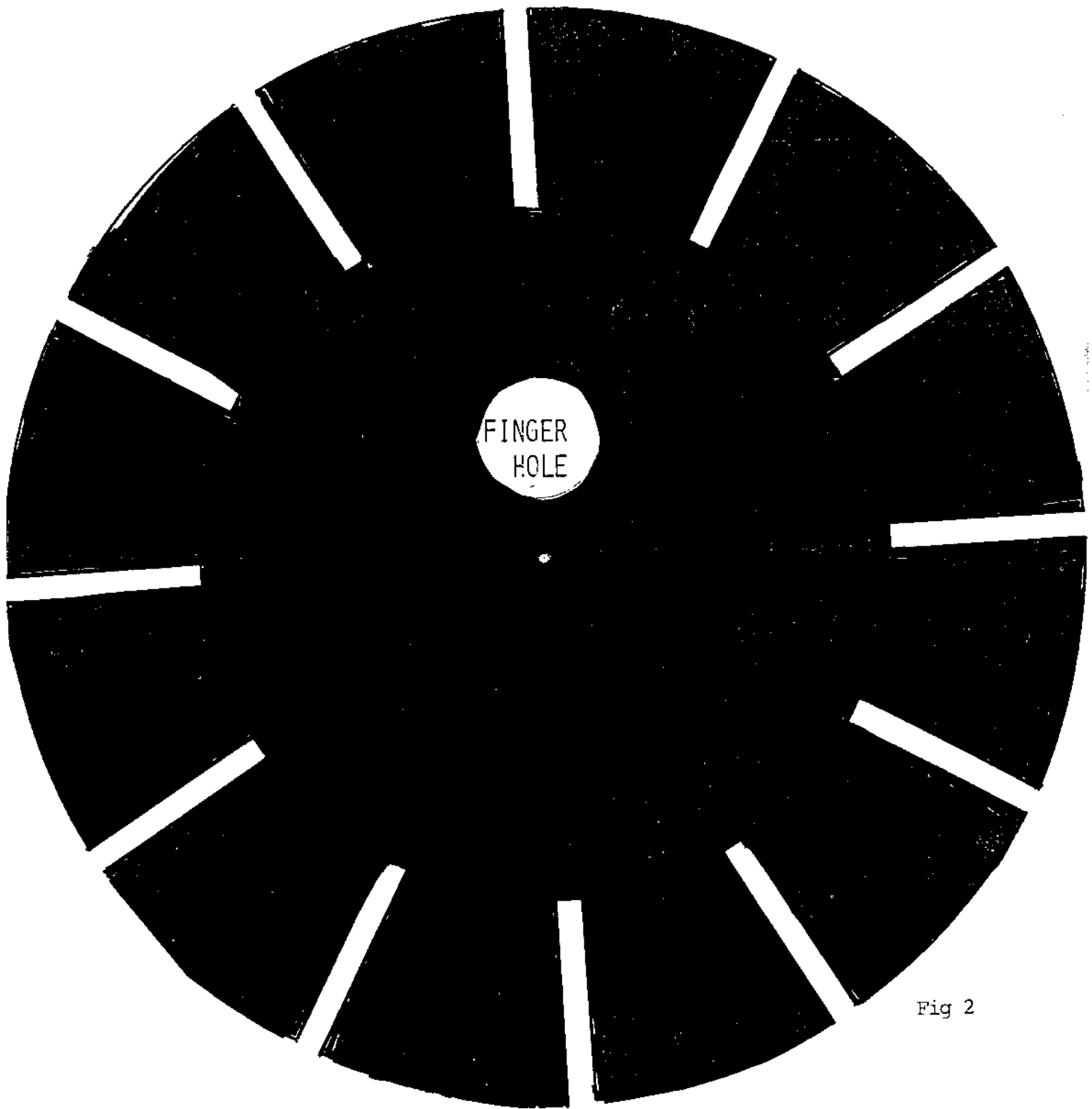


Fig 2

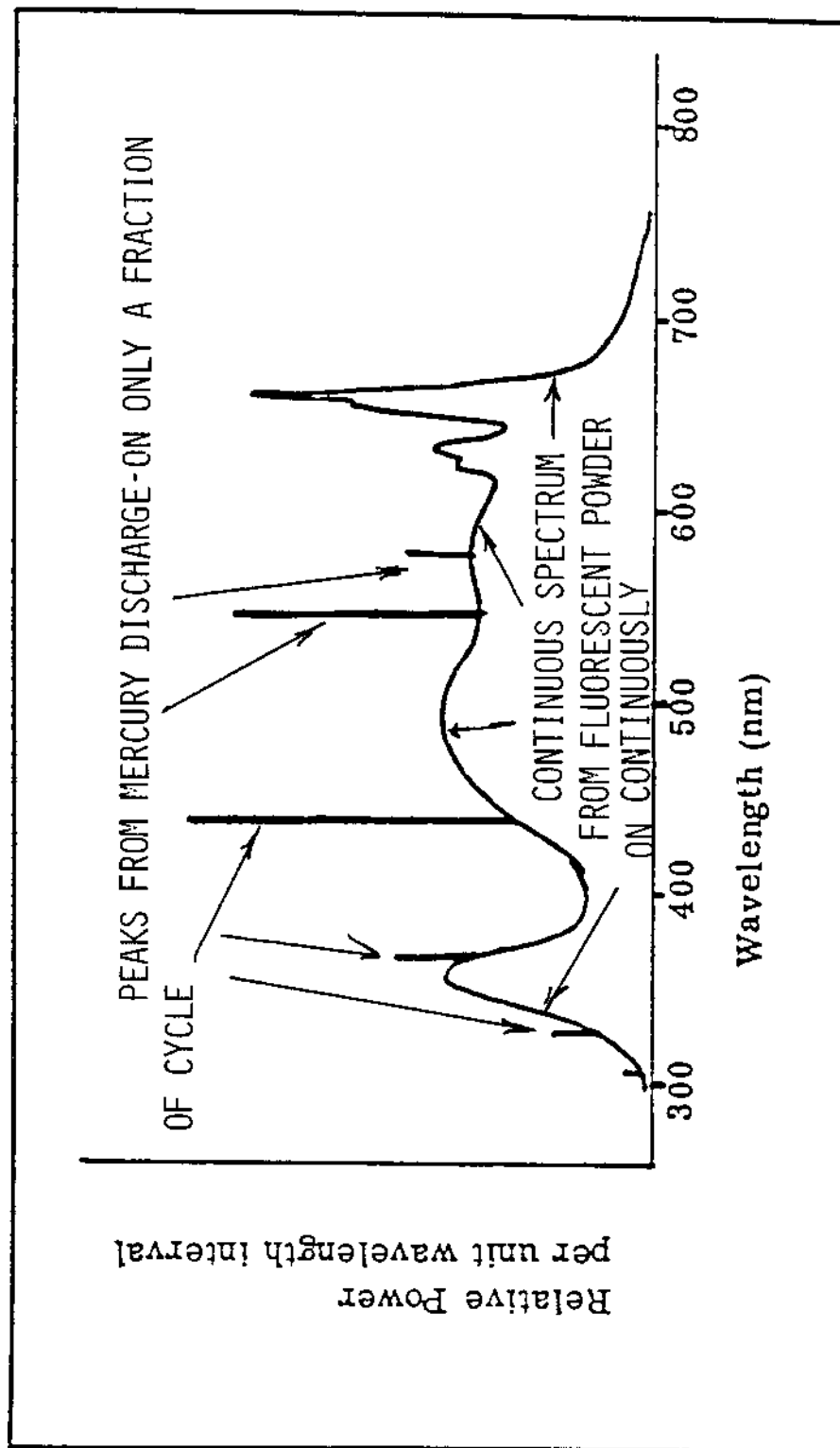


Fig 4

Spectral power distribution of a fluorescent tube (Artificial daylight, Thorn Lighting Ltd.)

Experiment 1.57 - SPINNING TOPS

The most intriguing aspect of angular momentum is the spinning top - and its many relations (tippetop etc.). The most obvious properties of the top are precession and nutation, and we must see how these are started by giving an impulse to the top. To make a top, cut a disk about four inches in diameter out of cardboard (or you can use the plastic top off a nut tin etc.), sharpen your shortest pencil (about two inches long) and push it through a hole, which you can start with a pair of scissors, exactly through the center of the disk but it must protrude less than an inch. Balance the top, if it needs it, by pushing paper clips evenly around the periphery of the disk, as shown. (Fig. 1) Try to get the center of mass as close as possible to the point about which the top spins. The additional mass of the clips will provide a greater moment of inertia, which will make the top spin better. The clips can be struck on with a little tape so they do not move once the top is balanced.

Now spin the top - unless you can spin it very fast, the axis of the top will slowly rotate about the point where the top touches the table. This is called precession. Spin the top as fast as possible, then blow vertically through a straw at a point on the edge of the disk, as hard as possible, as shown in figure 2. The top will not fall at the point where blown, but one side will slowly rise at 90° to this point and the opposite side will fall as shown in the figure. This gyroscopic torque is the basis for the toy gyroscope, the gyroscopic compass, and in guidance systems used in rockets. What makes it work? - A quantitative solution is given later, but qualitatively, blowing on the edge of the disk, together with the reaction where the top axis touches the table provides a torque trying to twist the top down where the air presses it, and giving the top angular momentum. But the top already

has a large angular momentum about its axis - and the vector representing this, which is like an arrow pointing along the axis rotates to put a larger component of angular momentum along the arrow representing the torque. Strangely enough, this moves the top in a direction perpendicular to that we would expect - i.e. to move down where the puff of air hits it - instead rotating the axis of the spinning top at right angles to the direction the air would appear push it.

Gravity acting on the center of mass of the top provides a similar torque - and again, the top moves at right angles to the direction we would naively expect - this is precession - the faster the top spins, and the larger the axis to the vertical, the faster the precessional rotation. Lastly, if we give a sharp puff of air - an impulse - to the edge of the top, it will bounce up and down as it precesses - as shown in figure 3 - this is called nutation. Even objects as large as the earth precess and nutate - but very slowly - in the precession of the earth, the axis take 20,000 years to make one rotation. Effects such as the growth of large mountains, which change the moment of inertia of the earth, can affect the rate of precession.

Quantitative -

The quantitative "why" of the spinning top depends on realizing that angular momentum can be represented as a vector - so a top spinning right handed seen from above is represented by a vector pointing downward along the axis, of magnitude $I\omega$ where I is the moment of inertia and ω is the angular velocity ($2\pi \times$ the number of rotations per second). Correspondingly, the torque, due to gravity, of magnitude $mg\ell$, shown in Figure 4, can also be represented as a vector pointing as shown. Now this torque must accelerate the top, and it does so by increasing the angular momentum by $G\delta t$ in time δt -

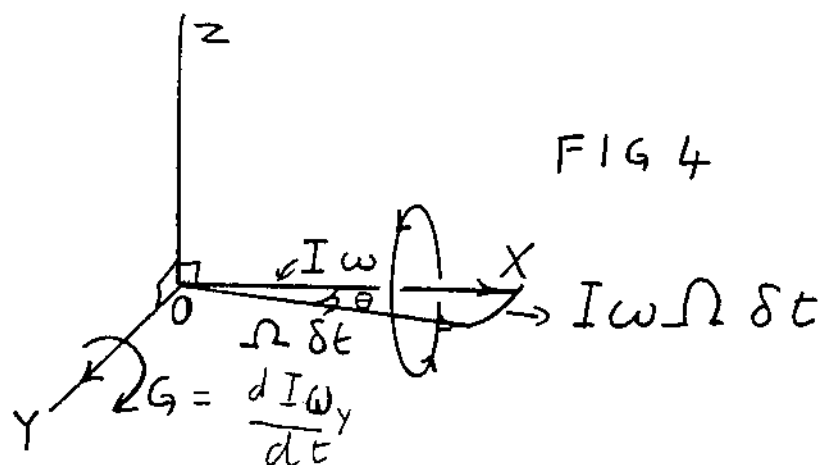
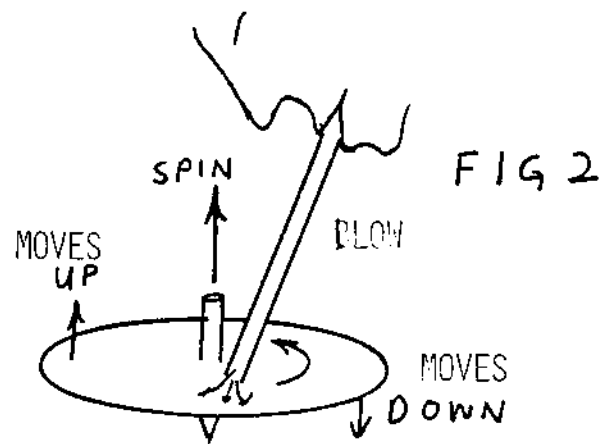
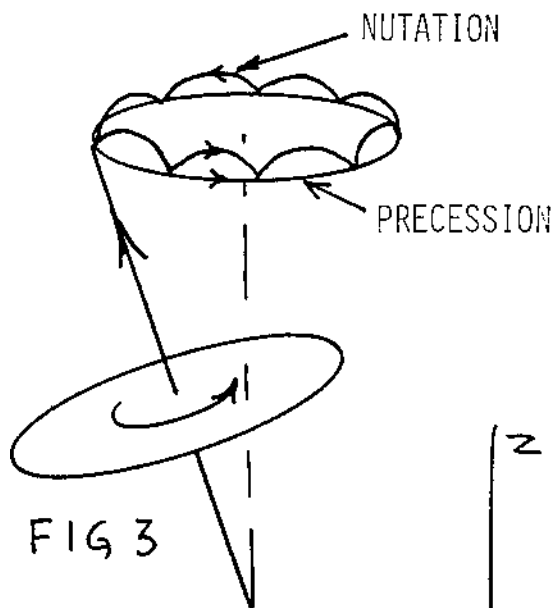
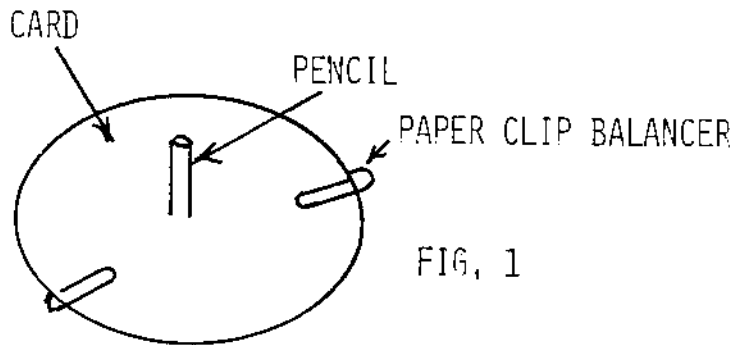
however, this change in momentum is actually accomplished by rotating the axis perpendicular to its length. Since the angular momentum is $I\omega$, the change in $I\omega$ in δt must equal $G\delta t$ - and this must occur in the direction of G - the y direction in the diagram.

$$\text{So } \delta(I\omega)_y = G\delta t$$

$$\text{From the diagram } \delta(I\omega) = (I\omega)\theta = (I\omega)\Omega\delta t$$

$$\text{and } I\omega\Omega\delta t = G\delta t$$

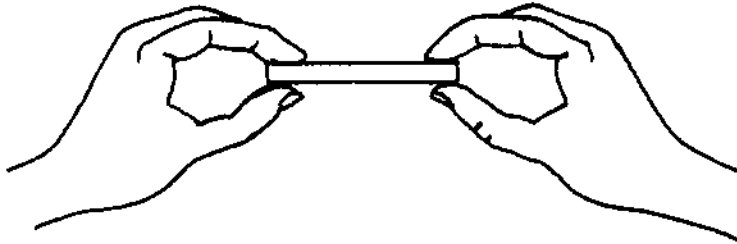
$$mg\ell = I\omega\Omega$$



Experiment 2.01 Properties of matter-stress and tension in chalk

Materials: - an unbroken piece of chalk

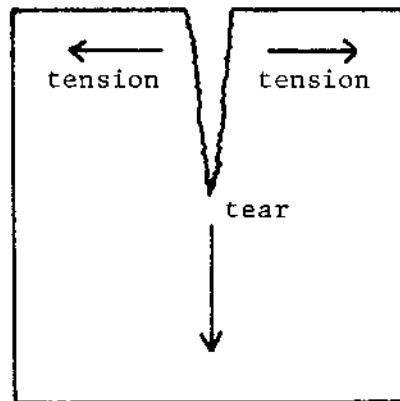
Take an unbroken piece of chalk, Grasp the opposite ends and pull, taking care not to twist or bend it.



The chalk will break cleanly across



This is because chalk, being a compacted dust, can easily withstand compression, but very little tension. However, a crack, once started in the chalk, propagates all the way across, as a sheet of paper tears: -

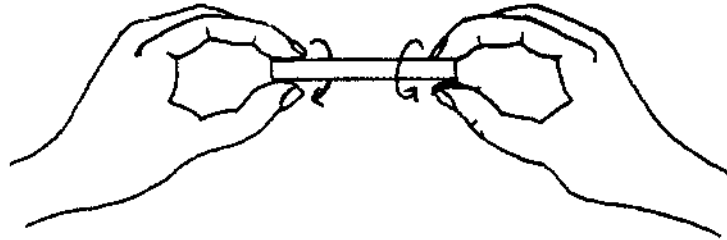


See Feynmann Lectures in Physics Vol. II p. 39-9.

Experiment 2.02 Properties of matter - torsional stress in chalk

Materials: - an unbroken piece of chalk

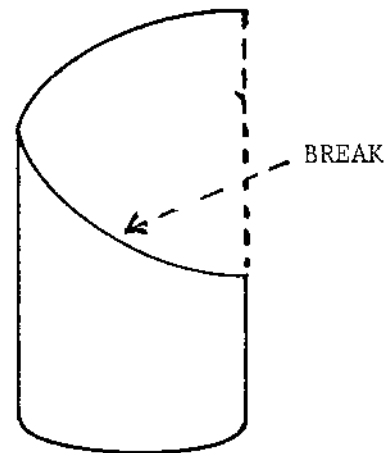
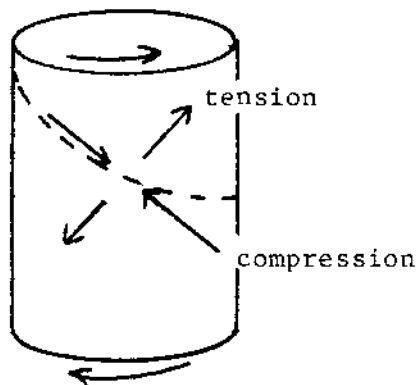
This is a continuation of the last experiment. Grasp the opposite ends of the piece of chalk, and twist without pulling or bending.



The chalk will break, with at least one portion showing a helical edge which is at 45° to the axis of the chalk.



This arises because the torsional stress strains the chalk in two directions at right angles.



One strain is a compression at 45° to the axis of its torque, the other is a tension, also at 45° to the axis. The chalk cannot withstand the tension, and breaks perpendicular to the tensile force.

Experiment 2.03 Surface Tension

Materials: - cup, two paper clips

Procedure: - Bend one paper clip as shown



Lay the other one across it

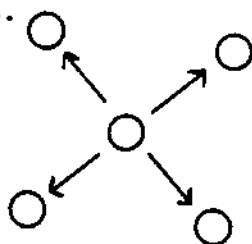


Fill the cup with water. Now, slowly lower the clip into the water, and, if done carefully enough, and provided the clip is perfectly flat, it will float. Rubbing the clip down the side of one's nose, to grease it, helps repel the water. The way in which the clip lies on the surface "skin" is clear to see - the surface dimples under the clip. Sometimes one or two clips may have to be tried, until one works, because of sharp edges.

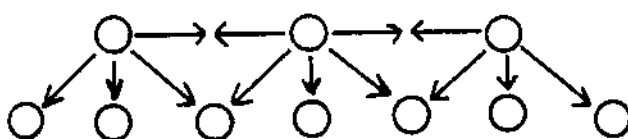
Qualitative Questions:

- (1) Why must you lay the clip gently on the surface?
- (2) Why would an aluminum clip work better than a steel one?

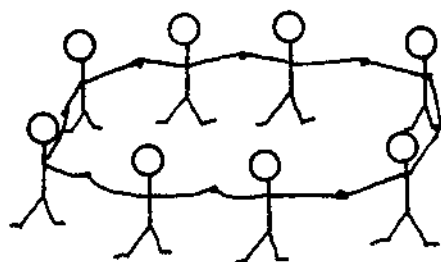
In the body of the liquid, the attractive forces between molecules act on the molecules in all directions.



However, in the surface, they can only act inward.



This forms a surface "skin", and the attraction between molecules in the surface pulls inward to hold the liquid in drops. If a group of people join hands in a circle, and pull together, they move inwards, in the same sort of way.



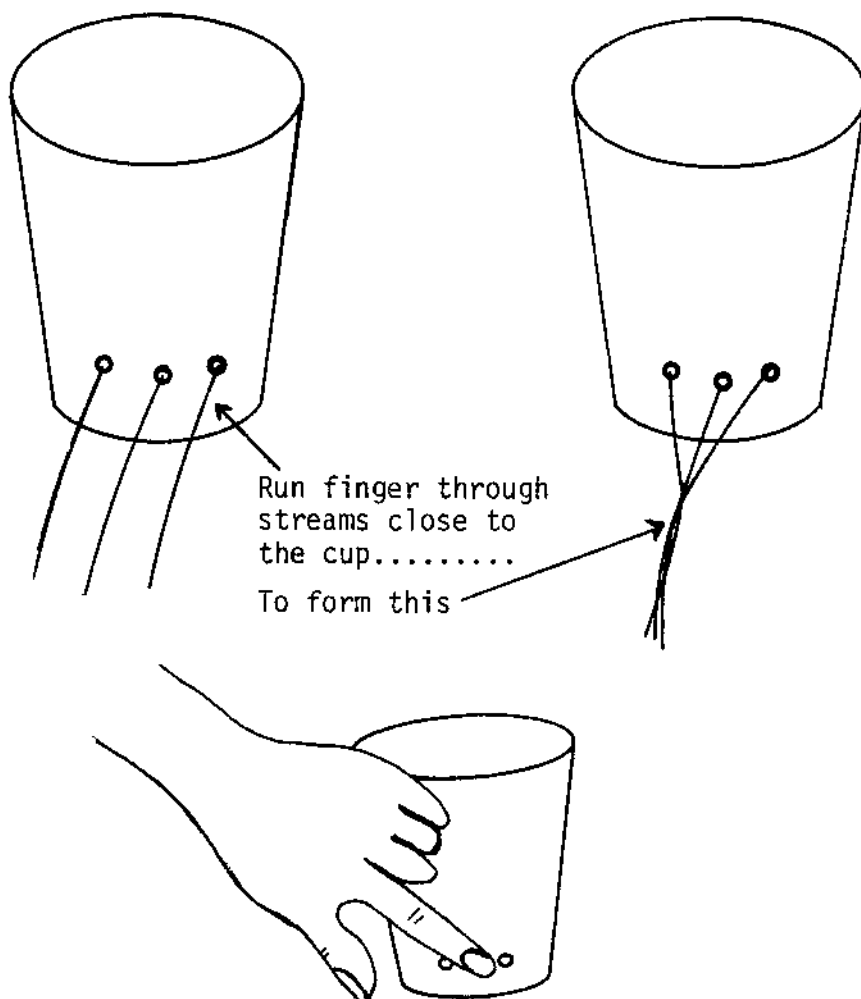
Experiment 2.04 Surface Tension in a liquid stream

Materials: Styrofoam (or paper) cup, sharp pencil

Procedure: Poke three small holes in the side of the cup, as close together and near the bottom as possible. Three streams of water will shoot out. Now run one finger close to the cup through the streams and note how they join together even after you've removed your finger.

Qualitative: What causes the streams of water to join together? Notice how they spiral about one another.

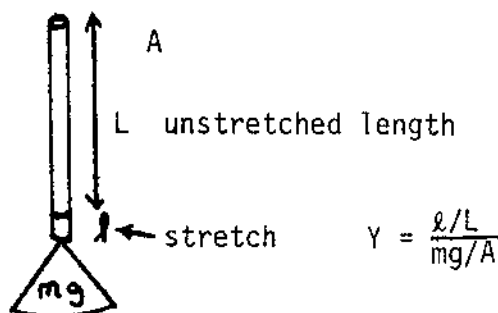
Quantitative: Can you estimate the surface tension force from the point where the streams join the size of the holes and the rate at which the cup empties?



Experiment 2.05 Youngs Modulus for Cellophane Tape

Materials: Cellophane tape, paper clip, cup, straw, marbles, paper clip

Procedure: The modulus of elasticity determines the change in shape of a body when a force acts on it - for example, if we hang a weight on a wire, it stretches, and the stretch is given by Young's modulus of elasticity. The modulus is defined as the stress, or fractional change in shape, divided by the strain, or force per unit area. So, for Young's modulus $Y = \frac{\text{stress}}{\text{strain}} = \frac{\text{fractional change in length}}{\text{force per unit area}}$



where L = unstretched length of wire

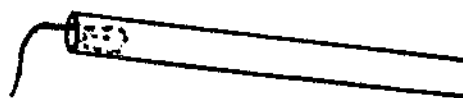
ℓ = extension of wire

Mg = force acting on wire

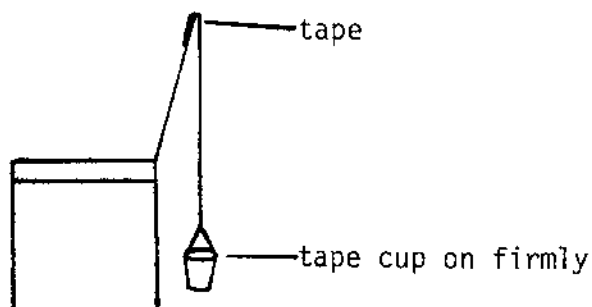
A = area of wire

We shall measure Young's Modulus for a piece of cellulose tape. The principal difficulty is measuring the rather small extensions involved. To do so we shall use a paper clip as a roller.

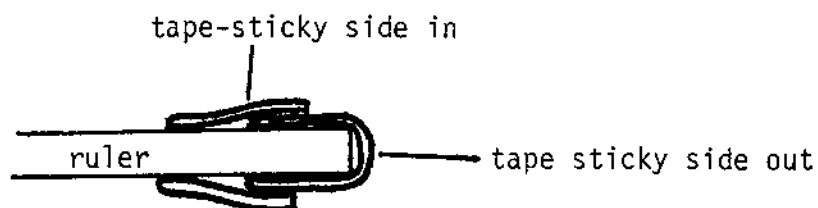
Open up a paper clip as shown, and push the closed end into a straw.



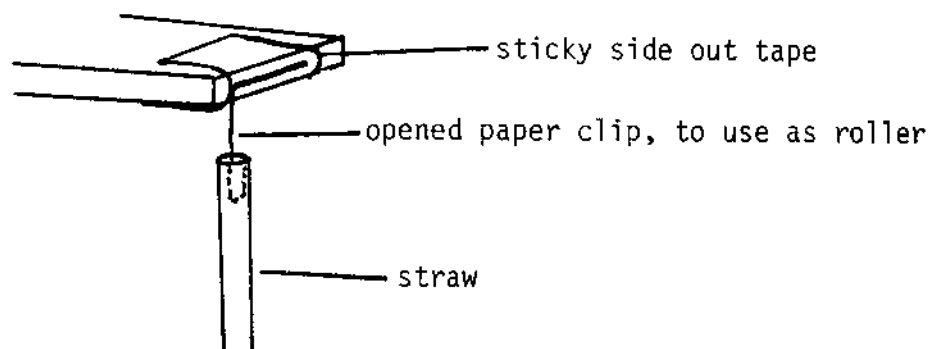
Hang a paper cup from a strip of tape over the back of a chair, edge of a table, etc. Make sure the tape passes over the edge



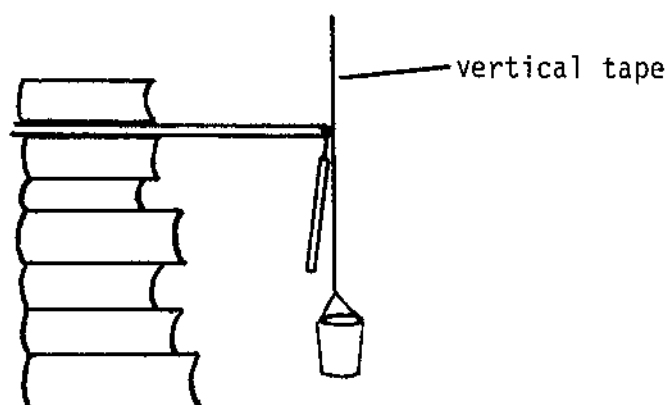
To measure the extension, attach a piece of tape, as shown, sticky side out, over the end of a ruler



Attach the opened paper clip to this, so

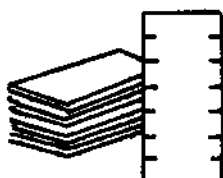


Now, place the ruler on a pile of books on the floor, and put some books on top to hold it steady. Then move the whole set-up so the roller paper clip sticks to the vertical hanging tape. One side of the roller should be on the tape on the ruler, the other side on the vertical tape. The two sticky tapes must not stick to one another



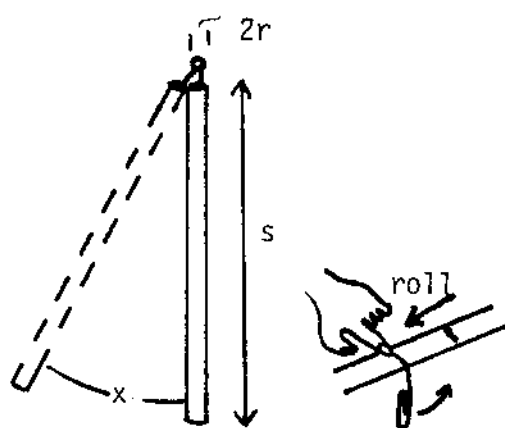
A thumb nail may be run across the roller to make sure the tape sticks. Now, as marbles are placed in the cup attached to the vertical tape, it stretches, rolls the paper clip, and the motion is magnified, so that the motion of the end of the straw is very large. Measure the movement of the end of the straw, using a ruler, or mark it on a sheet of paper to measure later.

To find the modulus of elasticity, we need to know the thickness of the tape. To find this, stick ten or twenty small pieces on top of one another, cut across, and measure the thickness with the ruler.



The area of the cross section of the tape is this thickness multiplied by the width.

The extension of the tape, Δ , is the distance moved by the end of the straw x multiplied by the ratio of the diameter of the paper clip wire forming the roller, $2r$ to the length of straw, $\Delta = \frac{2rx}{S}$ (S is the straw length)



The diameter of the wire may be measured roughly with a ruler. Otherwise, roll the paper clip, without the straw, along a strip of sticky tape, four or five times, and note how far it rolls. The distance is $2\pi r n$ where n is the number of rolls.

The length of tape L used extends from the support to the roller.

The mass of the marble can be found by weighing against a known quantity of water. The density of glass P_g is about 2.5, so the mass can be calculated roughly as $\frac{4}{3} \pi r^3 P_g$, measuring r with the ruler. (Generally marbles weigh about 5.4 gm.)

We now have all the quantities required to measure Y .

$$Y = \frac{\ell/L}{mg/A} = \frac{2rx/sL}{mg/Wt} = \frac{2rxwt}{sLmg}$$

W = width of tape

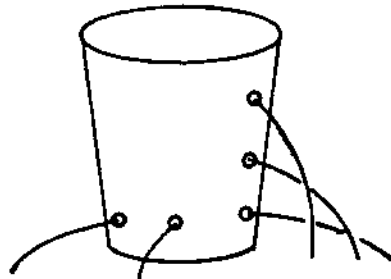
t = thickness of tape

Y is generally of the order 10^{10} dynes/cm². (10^{10} N/m²)

Experiment 3.01 Hydrostatics and Hydrodynamics - Hydrostatic Pressure

Materials: - Styrofoam (or paper) cup.

Procedure: - Poke small holes of the same size with a pencil in the cup at various levels, and around the cup at the same level as shown



Fill the cup with water. Note how the stream from the bottom hole shoots out much faster than that from the top. Also note how the streams from the holes at the same level always shoot the same distance, even though that distance changes as the water level in the cup drops.

Qualitative: - The pressure forces the water through the holes. Since it goes further, the pressure is largest at the bottom, smallest at the top, and since, at the same level, the holes all shoot the same distance, the pressure is constant a constant distance from the top. Pressure acts in all directions at any one point in the liquid

Quantitative: - The pressure is the force per unit area, and is given by

$p = \rho gh$ where ρ = density of liquid

g = acceleration due to gravity, L = the

length to the point where the pressure is measured.

From the equation for the velocity of a particle

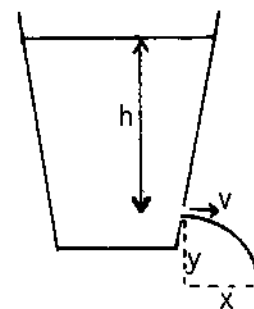
falling a distance h , the velocity of the stream

is given by $2 gh = v^2$

If the stream projects horizontally

$$x = vt, \quad y = \frac{1}{2} gt^2 = \frac{1}{2} g \frac{x^2}{v^2} = \frac{1}{2} \frac{x^2}{2h} = \frac{x^2}{4h}$$

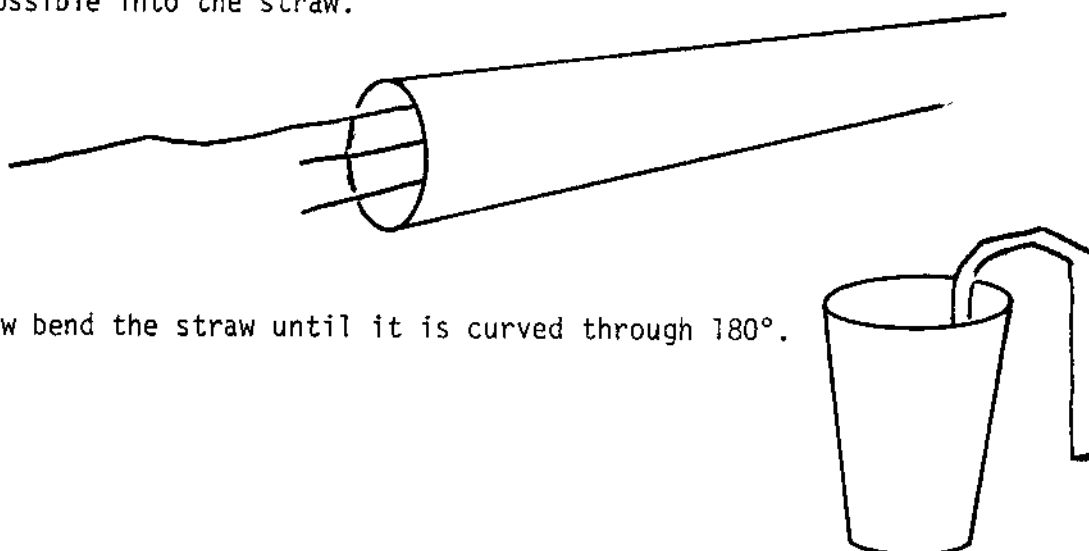
You can measure x , y and h . Is the formula correct?



Experiment 3.02 The syphon

Materials: soda straw, cup

Procedure: The difficulty is to bend the straw, as shown, without breaking or kinking it too much. Unfold three paper clips, and push them as far as possible into the straw.



Now bend the straw until it is curved through 180° .

Fill the cup with water, place the syphon in as shown and suck the open end. The water will flow uphill, down the far side, and continue to run until the cup is empty.

Qualitative: Why does the water flow uphill? Why is the flow so slow?

Quantitative: How high could the syphon go? Could you syphon over a mountain?

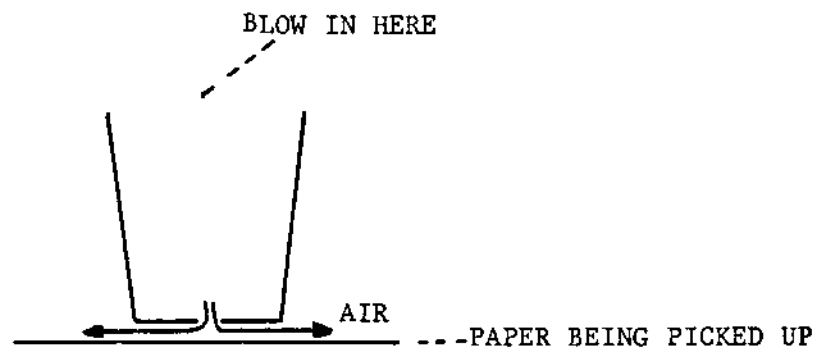
Because the syphon is very narrow, it can be caused to start on its own by tipping until the water is just below the lip. What makes the water flow uphill without suction?

Note: Running very hot water over the straw helps it to bend.

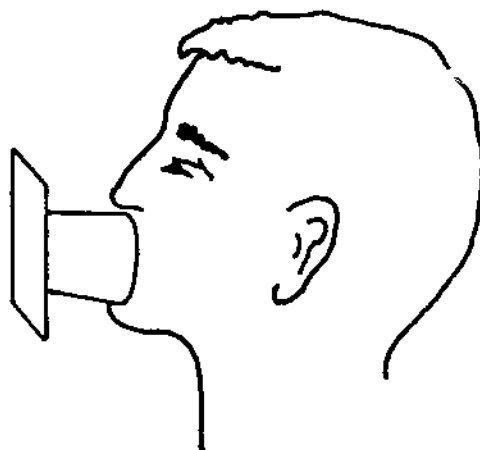
Experiment 3.03 Bernoulli Effect

Materials: - Styrofoam cup, and paper

Procedure: - Bore a hole about 1/4" in diameter in the bottom of the cup. Place over the sheet of paper. Place the cup over your mouth, and blow into it. You should feel a stream of air coming out of the hole. Place the cup over a sheet of paper, and blow. The sheet may easily be picked up, rather than being blown away, because of the low pressure rapidly moving air.



A straw through a cotton reel will do the same thing, but a Dixie cup with its raised bottom will not work as well. The reason being air must flow through a narrow space over a large area between the paper and the flat bottom of the cup, to provide sufficient force to pick up the paper.



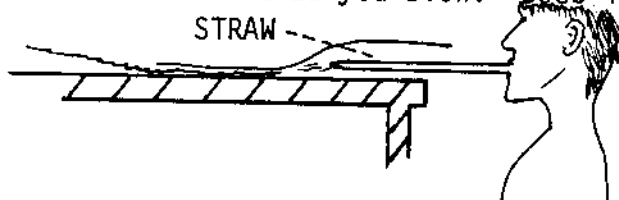
Experiment 3.04 Bernoulli's Principle

Materials: - A sheet of paper, and a straw

Instructions: - Place the paper (8 1/2 x 11) over the straw at the edge of the table. Blow. Does the paper blow away, as you might expect? The end of the straw under the paper should be flattened.

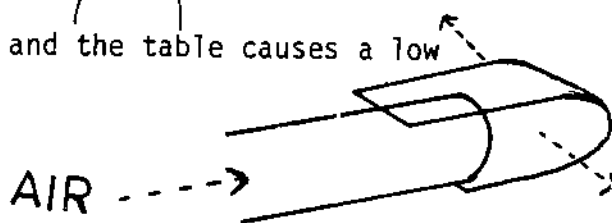


Try peeling the paper off the table as you blow. Does it tend to stick?

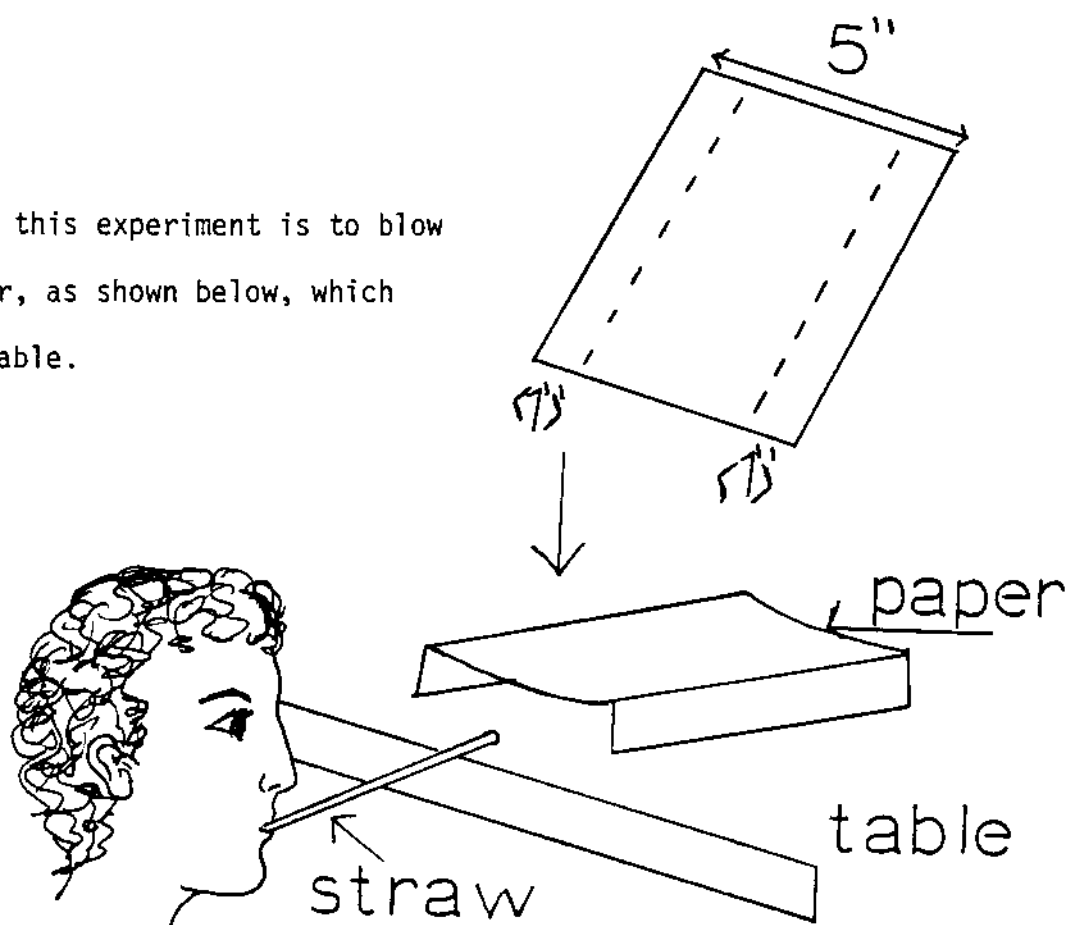


The rapid flow of air between the paper and the table causes a low pressure area here.

Note: If you have trouble with this experiment, stick a piece of tape across the end of the straw, and try again.



A slight variation on this experiment is to blow under the folded paper, as shown below, which sucks down onto the table.



EXPERIMENT 3.05 The Atomiser

Materials: Straw, cup

Procedure: Either cut the straw partially through, as shown, or completely through, and tape the two halves to a sheet of card.



Put one end into a cup of water, and blow through the other end. The suction is small, so fill the cup quite full of water, and ensure the top of the vertical straw is near the surface.

Qualitative Question: What happens?

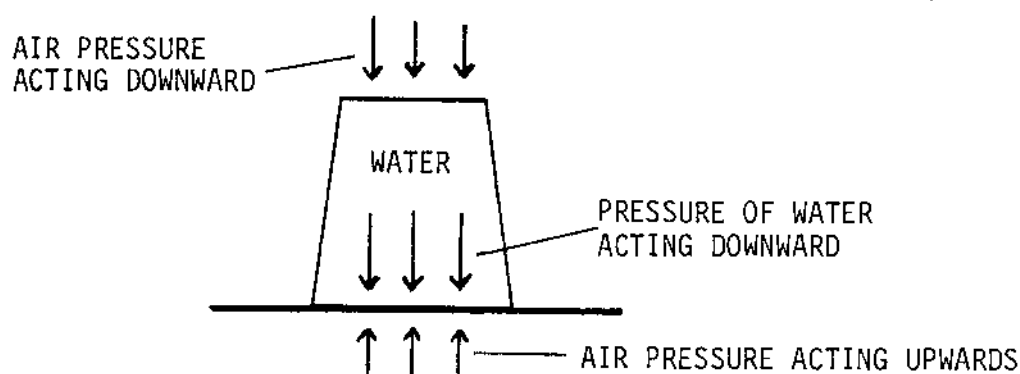
Bernoulli's principle tells us that, when the speed of the air is high, the pressure is low. Here, the speed is high just above the straw dipping into the water, and the low pressure sucks it up the straw, allowing it to be sprayed "atomized". Sometimes this suction is not very great and the straw has to go deep into the liquid.

Experiment 3.06 Atmospheric Pressure

Materials: - cup, paper, water.

Procedure: - Fill the cup to the brim with water, so that the meniscus stands high. Place a sheet of paper or card on top, and smooth it down so that there are no air bubbles, and the paper lies smoothly across the top, touching the brim. Smoothly, but rapidly turn the cup upside down. Suprisingly, the water does not run out. You may require two or three times to get it right, and it may be necessary to hold the paper on until upside down.

Qualitative: - The atmosphere presses down on us from all sides, not merely from above. When the water presses down on the paper, the pressure



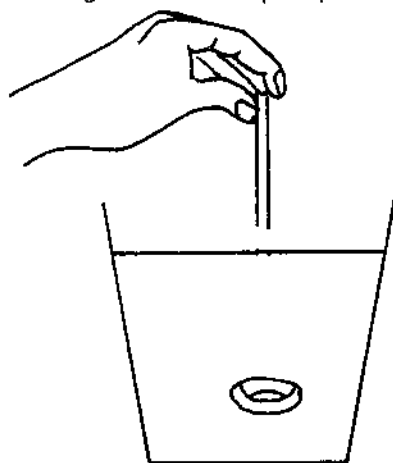
of the atmosphere pushes it back, and it cannot run out.

Quantitative: If we built a cup so high the pressure of water when inverted were greater than air pressure, the water would run out. How tall must such a cup be?

Experiment 3.07 Vortex Rings

Materials: - Cup, colored water or ink, (Grape Soda works Fine),
soda straw.

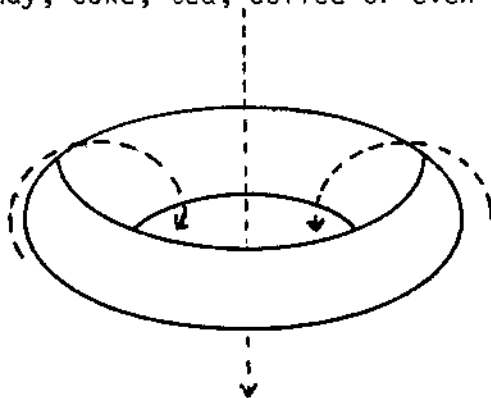
Instructions: - Fill the Cup with water, suck up a little ink in the soda straw, and pinch it off. Place the open end of the straw about one inch above the water in the cup, and give a sharp squeeze in the middle, rapidly expelling some ink.



A vortex ring shoots out, and spreads out as it strikes the bottom of the cup. Some practice and expertise is required to get this experiment to work (as with blowing smoke rings) but the flow of liquid out of the tube is always interesting to follow.

Qualitative: - A vortex involves circulation of an annulus of fluid. It cannot be generated in a non-viscous fluid, but once formed, is highly stable, persisting for long periods of time.

Note: If no ink is handy, coke, tea, coffee or even milk will do.

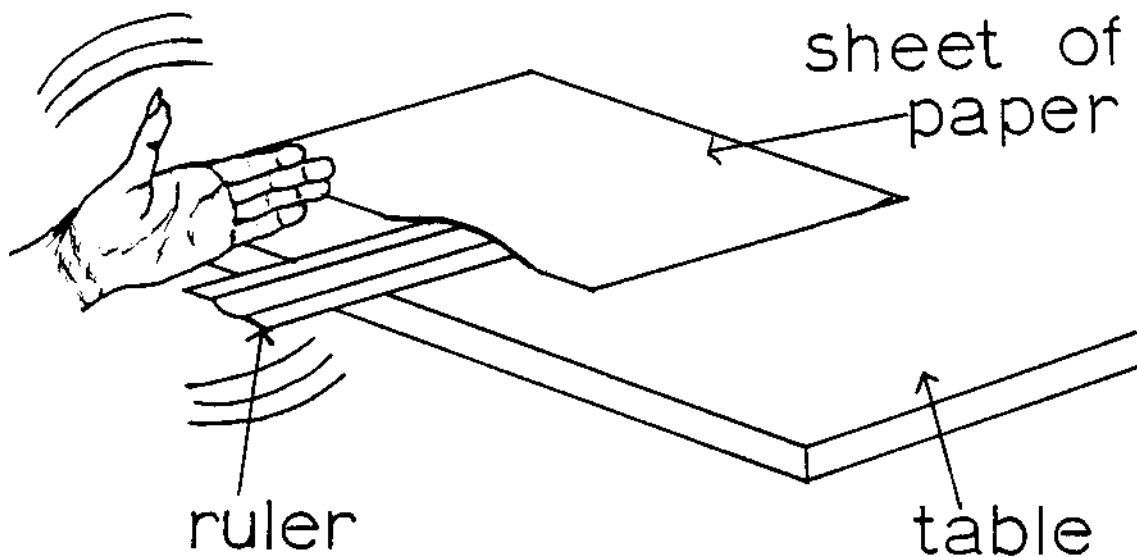


See: "The Flying Circus of Physics", 4.74, Jearl Walker (Wiley).

Experiment 3.08 Pressure of the Air

Materials: Sheet of paper, ruler.

Procedure: Lay the ruler over the edge of the table, and place the sheet of paper on top. The ruler may easily be dealt a blow capable of breaking it (either use an old ruler, or don't hit so hard!) even though held down only by the paper.

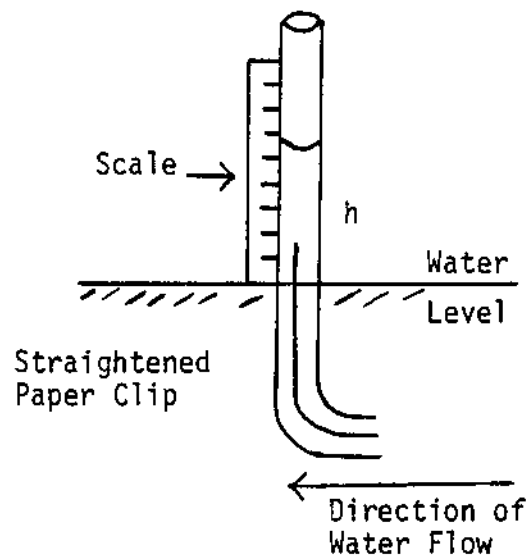


Why is this? The reason is that the ruler pushes up on the paper when struck, attempting to create a vacuum underneath it. Since there is no time for air to rush into this vacuum, the atmospheric pressure on top of the paper holds the ruler down so strongly that the ruler breaks.

Experiment 3.09 Sail boat log

Materials: Soda straw, paper clip.

Procedure: Straighten a paper clip, and insert it into a straw as far as it will go. Bend the combined clip and straw through a right angle, and cut out and attach the scale over as shown.

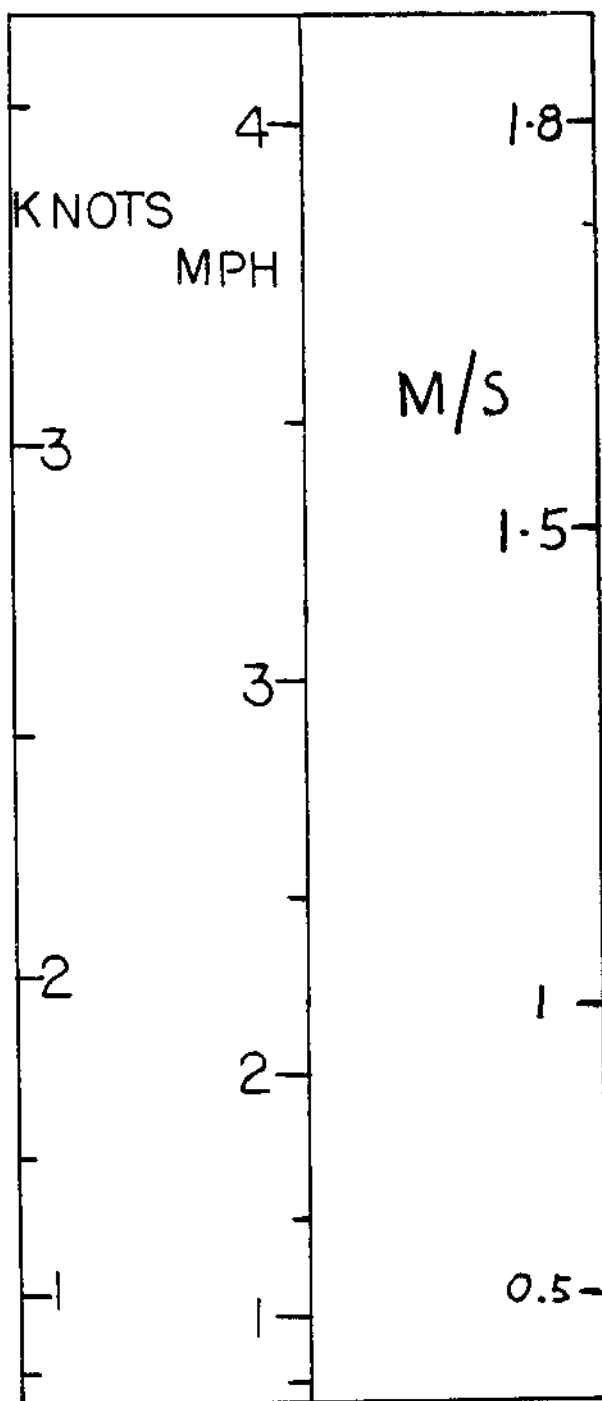


Insert the log in the water over the side of the boat, so the open end faces forward, and the zero of the scale lies in the surface of the water. As the boat moves forward, water will rise inside the straw. Bernoulli's theorem (or simple conservation of energy) tells us that

$$gh = \frac{1}{2} v^2$$

where h is the height the water rises in the tube, and v is the speed of the water past the open end of the tube. The scale calibration can be checked by a conventional old fashioned log - a piece of wood attached to a string.

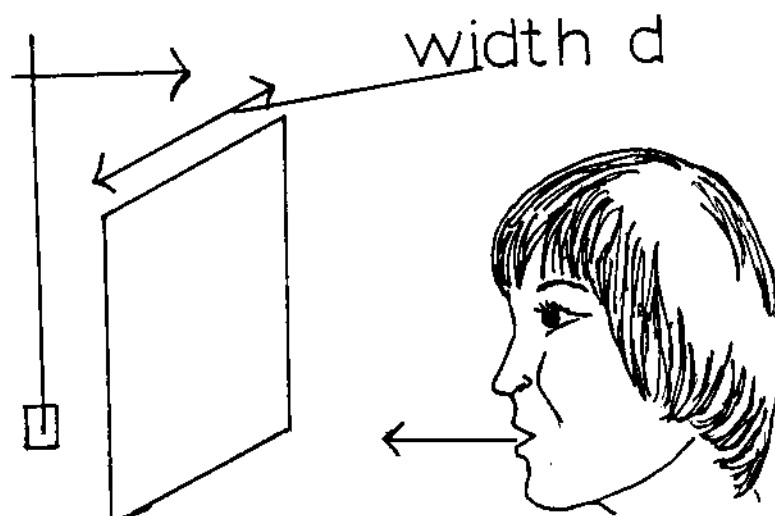
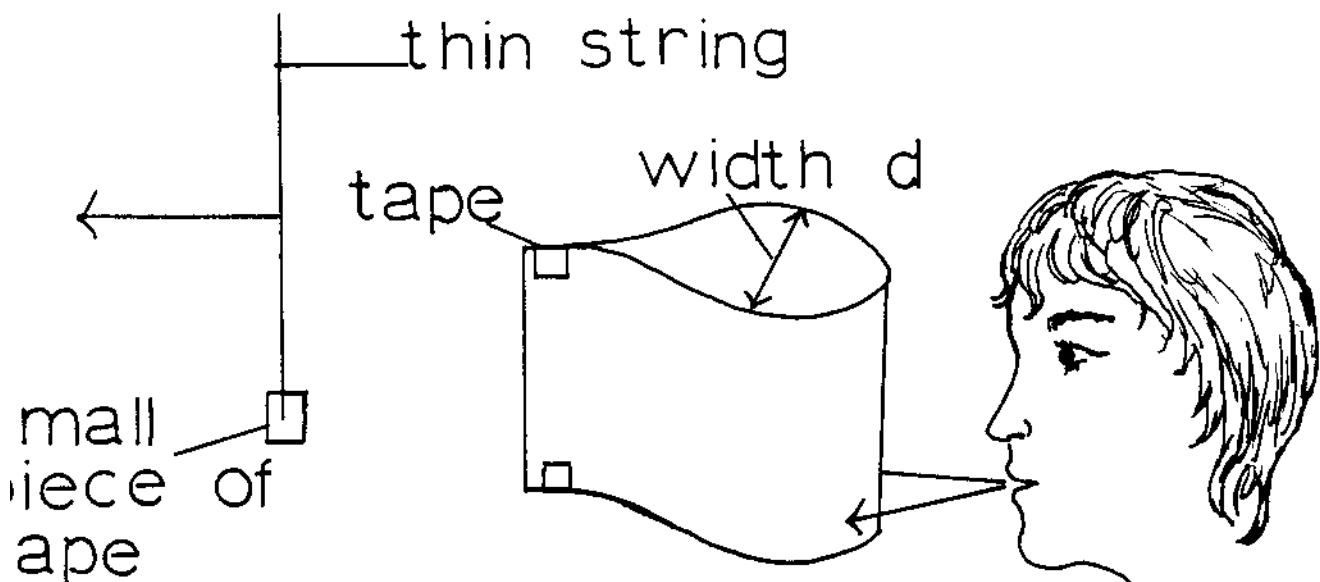
One knot is tied in the string every 50 ft. The log is thrown overboard and the number of knots in 30 seconds counted. A knot is a nautical mile per hour; or 1.15 mph.



Experiment 3.11 Streamlining of Aerofoils

Materials: Paper, tape, string.

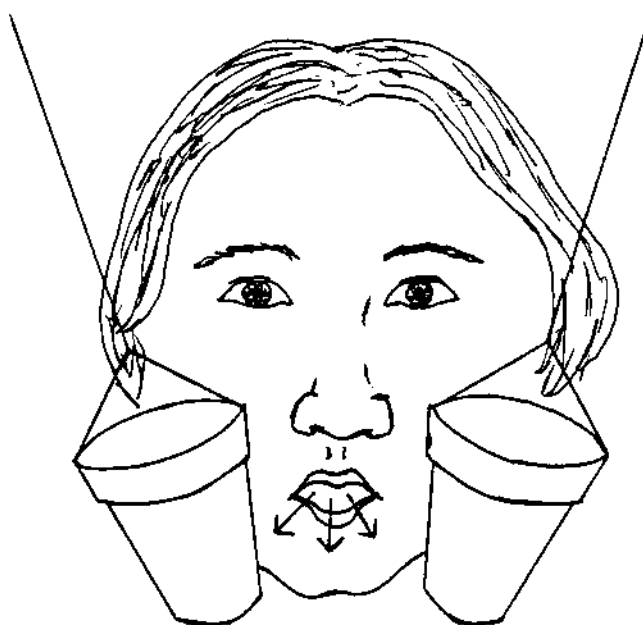
Procedure: The flow of air over objects is of vital importance - not only for aircraft, but to stop windows being sucked out of tall buildings, etc. Fold a sheet of paper into an aerofoil shape by taping the ends together as shown. Blow over it, and use a small piece of tape on a thread or piece of thin string to show the flow of air. Now replace the airfoil section with a sheet of paper or card the same width. Notice your detector does not move, or moves towards the card showing the motion of the air is as shown below



Experiment 3.10 Bernoulli Effect

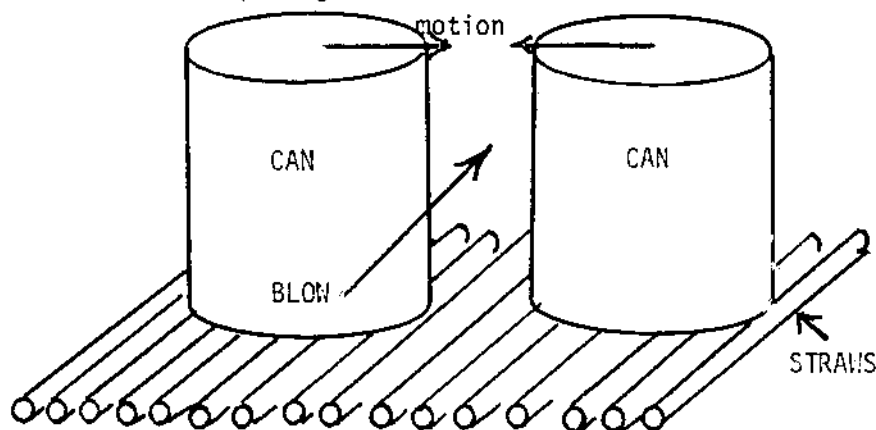
Materials: Paper cups, straw, sheet of paper, string

Procedure: Hang two cups on strings, as shown, close to one another. You can hold the strings by hand, or off a table. Blow between them, and notice how they are drawn together.



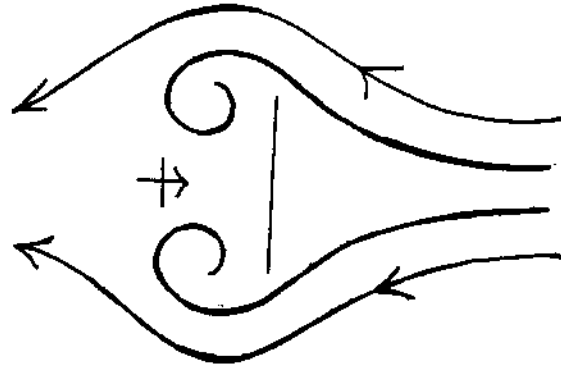
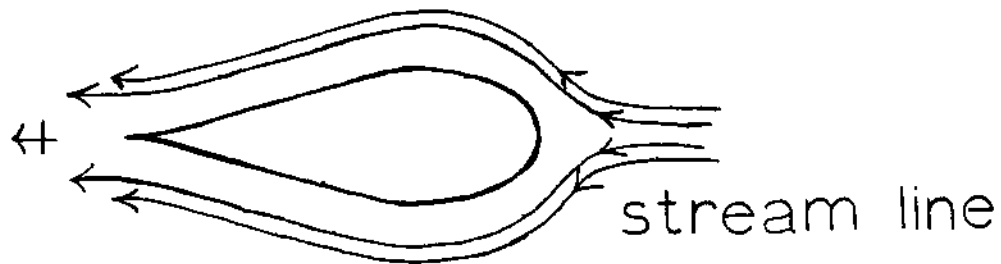
Try to explain why.

Explanation: The air moves rapidly between the two cups, and Bernoulli's principle tells us that where there is a high velocity, the pressure is low, sucking the cups together.

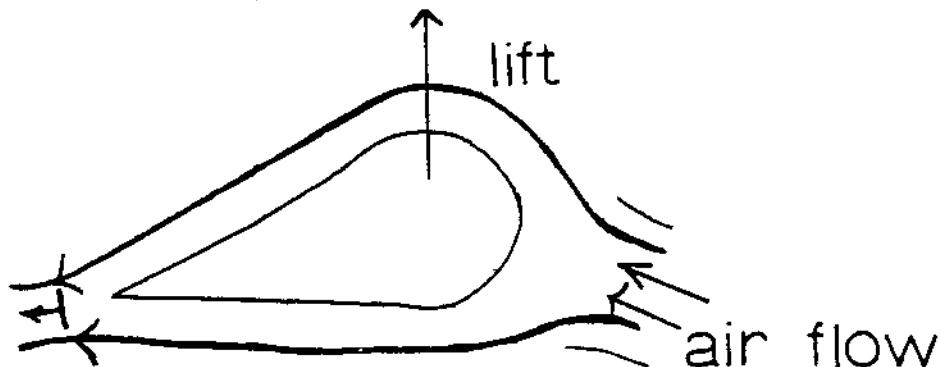


In his book "Invitations to science enquiry",* Tik L. Liem has a nice variation on this experiment, where you blow between two empty aluminum cans supported, as shown, on drinking straw rollers. The cans roll together.

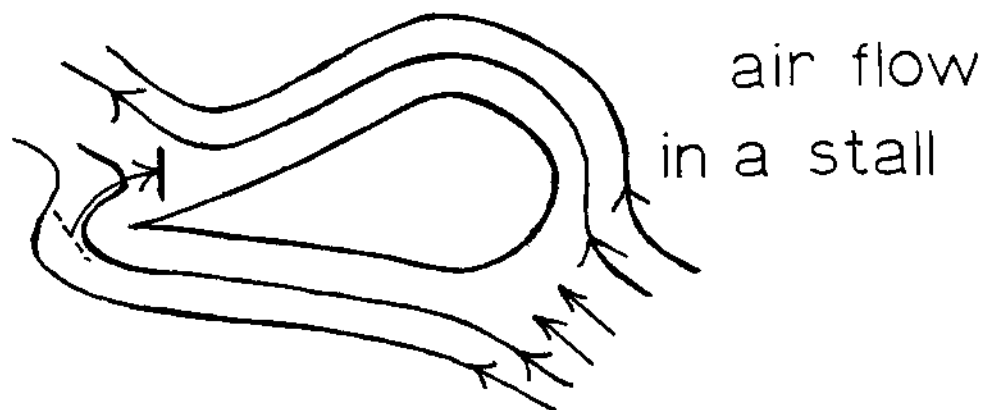
* Obtainable from :- Dr. Tik L. Liem, St. Francis Xavier University, Antigonish, Nova Scotia, Canada, B2G 1C0



The airfoil is a wing shape, and you can use it to investigate stalling, which used to be a prime cause of aircraft crashes. Turn the foil as you blow over it, as shown. You will notice a lift produced by the faster motion of the air on top of the wing. But if you twist further, very



suddenly the flow detector will be sucked on top of the wing, as the air ceases to flow smoothly over the trailing-edge, the lift ceases, and, if it were a plane, it would suddenly dive.



Experiment 3.12 The Flight of a Baseball or Golf Ball

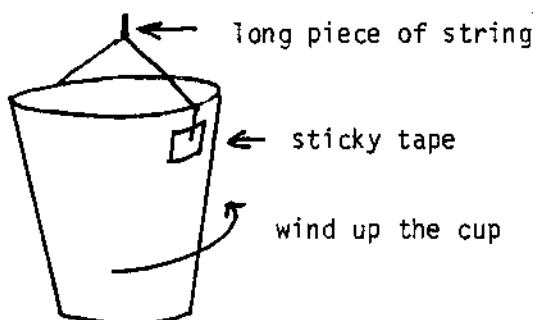
Equipment: Cup, string, marbles, scissors

Procedure: We can use a paper dirigible to show how the flight of a ball is affected by spin. Cut out the strip of paper (figure 1a) making a slot at one end and a tab at the other. Slip the tab through the slot, and hold the strip in the middle, so that it forms a loop as in Figure 1b. Hold the loop as high as possible, and drop it. It will rotate rapidly, about a horizontal axis, falling slowly to the ground.

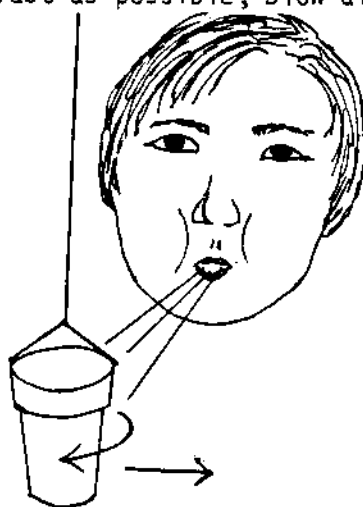
Which way does the loop drift as it falls? You will find the loop drifts bodily in the direction the lower part is traveling, whichever way that is, as shown in Figure 2.

If the dirigible rotates sufficiently rapidly we can regard it as a continuous surface, and we see from the figure that as it falls, air moving past the rising right hand side of the dirigible will be accelerated slightly, moving faster, whereas that traveling past the downward moving left hand side will be slowed. Bernoulli's principle tells us that the pressure will be slightly less in the region of faster moving air, hence the dirigible, in addition to falling, will be sucked to the right. Were it to revolve the other way, it would move to the left. It is for a similar reason, of course, that we put spin on a golf ball to encourage it to stay aloft. We are assuming the effect of air inside the rotating loop can be neglected.

Another way of demonstrating the same principle is to attach a fairly long piece of string to a cup as shown,



and hang it from some fairly high place, or have a friend hold it. Now, weight the cup with one or two marbles, and wind it up, so that it spins on release. When it is spinning as fast as possible, blow directly at the cup.



TAB →

FIG 1A

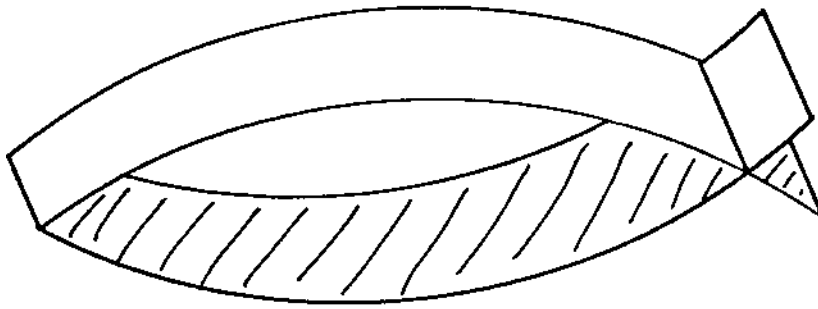
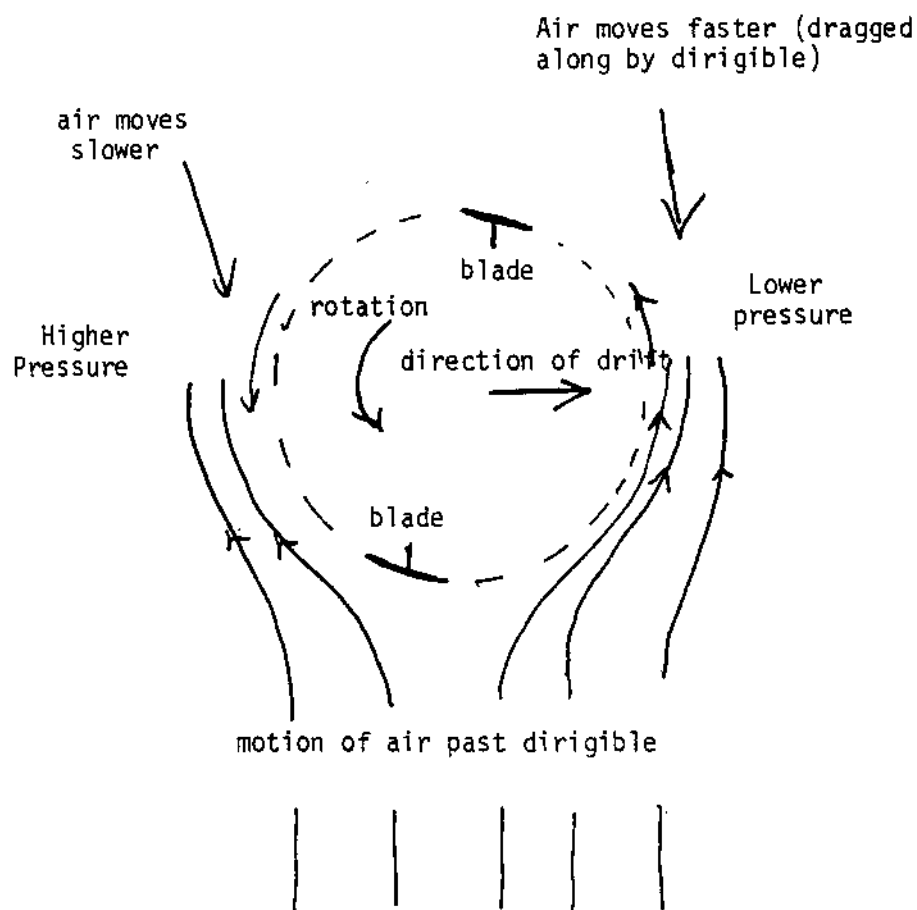


FIG. 1b



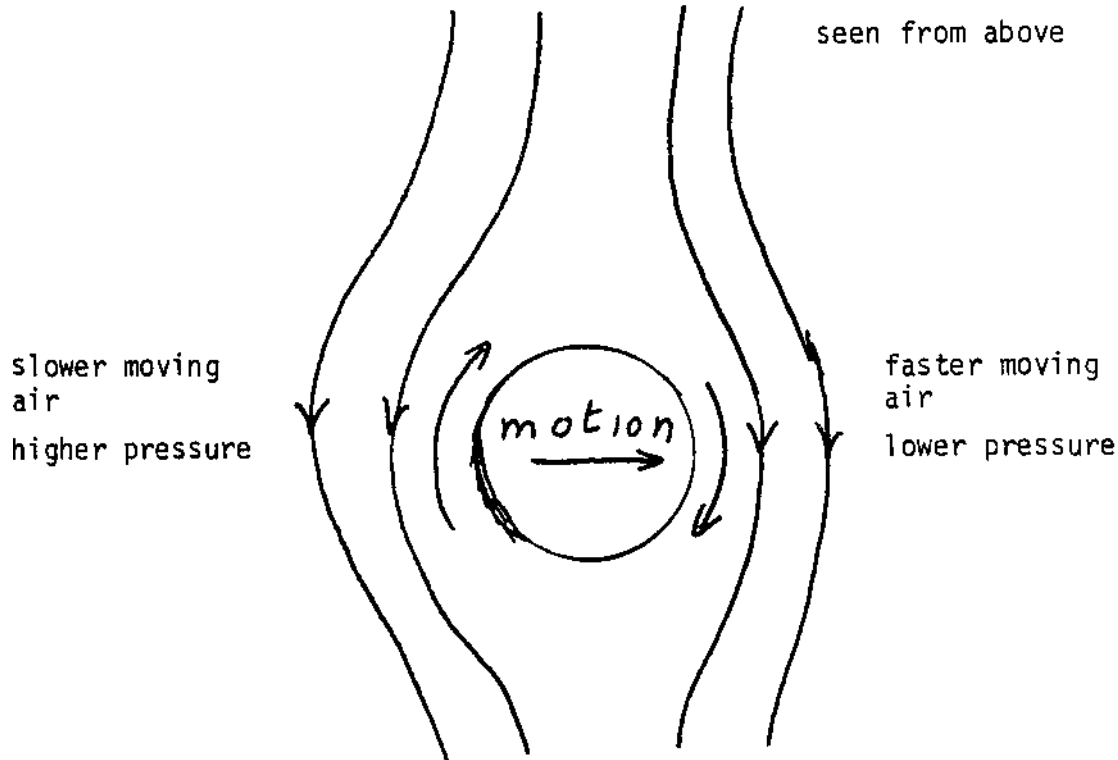
FOLD →

Fig. 2

SLOT →

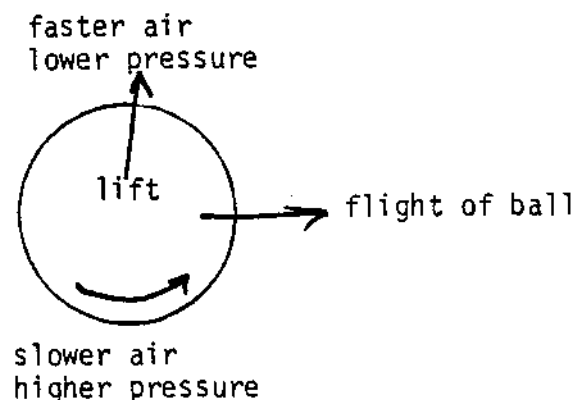
If the cup is spinning clockwise, seen from above, as shown, it will not be blown directly backwards, but should also move towards the left. If the cup spins anticlockwise, it should move to the right. Do you find this is so? Can you explain this in terms of Bernoulli's principle?

Explanation: Looking down from above, the spinning cup drags the air with it on one side, and the high air velocity produces a lower pressure, according

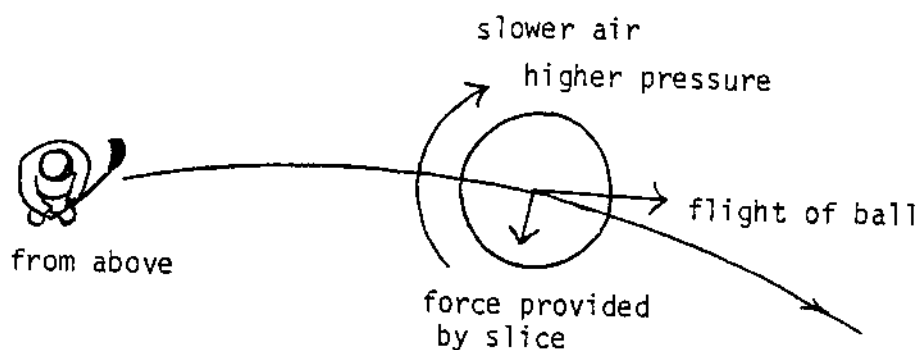


to Bernoulli's principle. Contrariwise, on the opposite side of the cup the air is slowed down by the spinning cup, producing a higher pressure. The cup will move towards the region of low pressure.

This effect finds application in the flight of a golf ball, which is given bottom spin to make it stay aloft. The dimples on the ball help drag the air round with it.

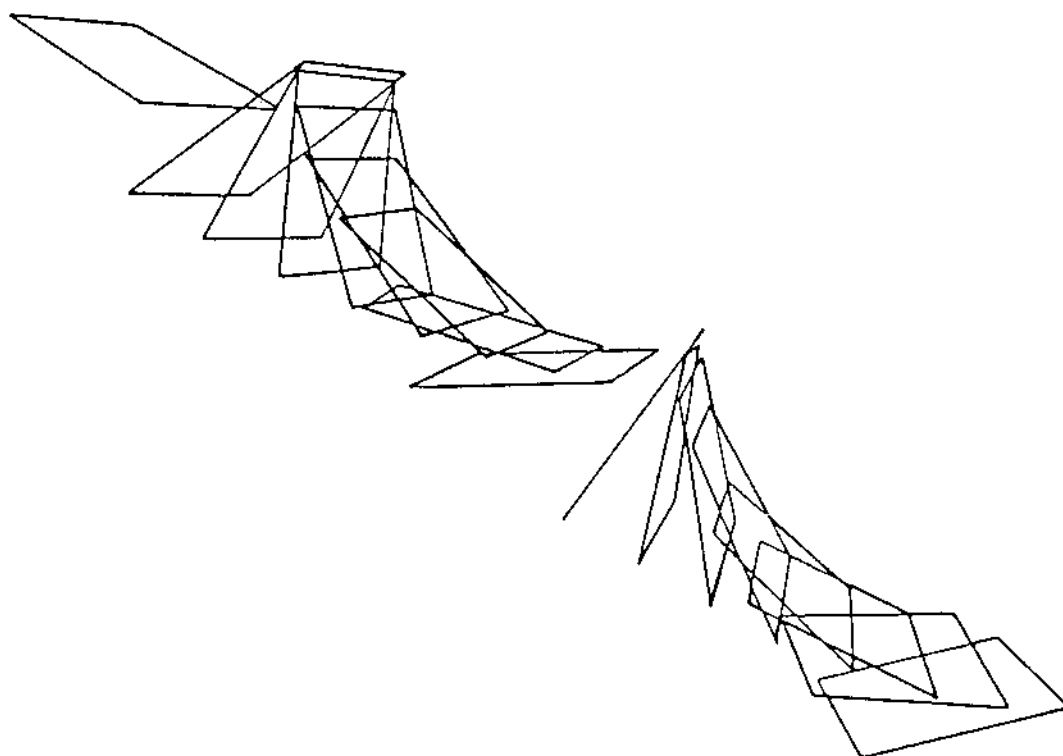


Correspondingly, if the ball is "sliced", i.e. given a clockwise spin seen from above, by a right handed player, it will curve away from the player. The situation where the ball curves away from the player is shown below.



A baseball will curve away from the pitcher, if spinning clockwise seen from above, for the same reason.

Even more simply, take a flat card - a playing card, visiting card 4x5 card or computer card will do, and drop it. It will rotate, and drift in a specific direction because of Bernoulli's principle, as shown in the figure, taken with a strobe light.



Stroboscopic photograph of a falling computer card, 50 flashes/c. Note the rapid rotation to the position beyond the horizontal.

Experiment 3.13. Levitating a Dime

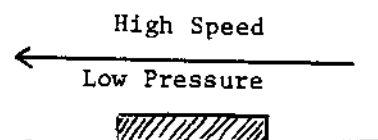
It is truly amazing how many qualitative and quantitative physics experiments can be done with very simple equipment if you put your mind to it. There are many curious and paradoxical problems which can be solved using Bernoulli's principle. One is to get a dime off a table into a cup on the table without touching the coin. The cup must be shallow, or tilted so that the lip is about 2 cm off the top of the table, and about 2-3 cm from the edge of the table, as shown. It looks impossible for the dime to get into the cup, but if you blow hard and suddenly, parallel to the table top, the dime hops in. As the air is blown rapidly over the top of the dime, Bernoulli's principle tells us that the pressure is lowered there, and the pressure differential between the top and bottom of the dime raises it off the table, and allows it to be blown into the cup. So much for the qualitative explanation, which will satisfy most people, but suppose we apply a little mathematics. Bernoulli's principle states that the pressure differential, p , between the top and bottom of the dime is given by

$$p = \frac{1}{2} \rho V^2$$

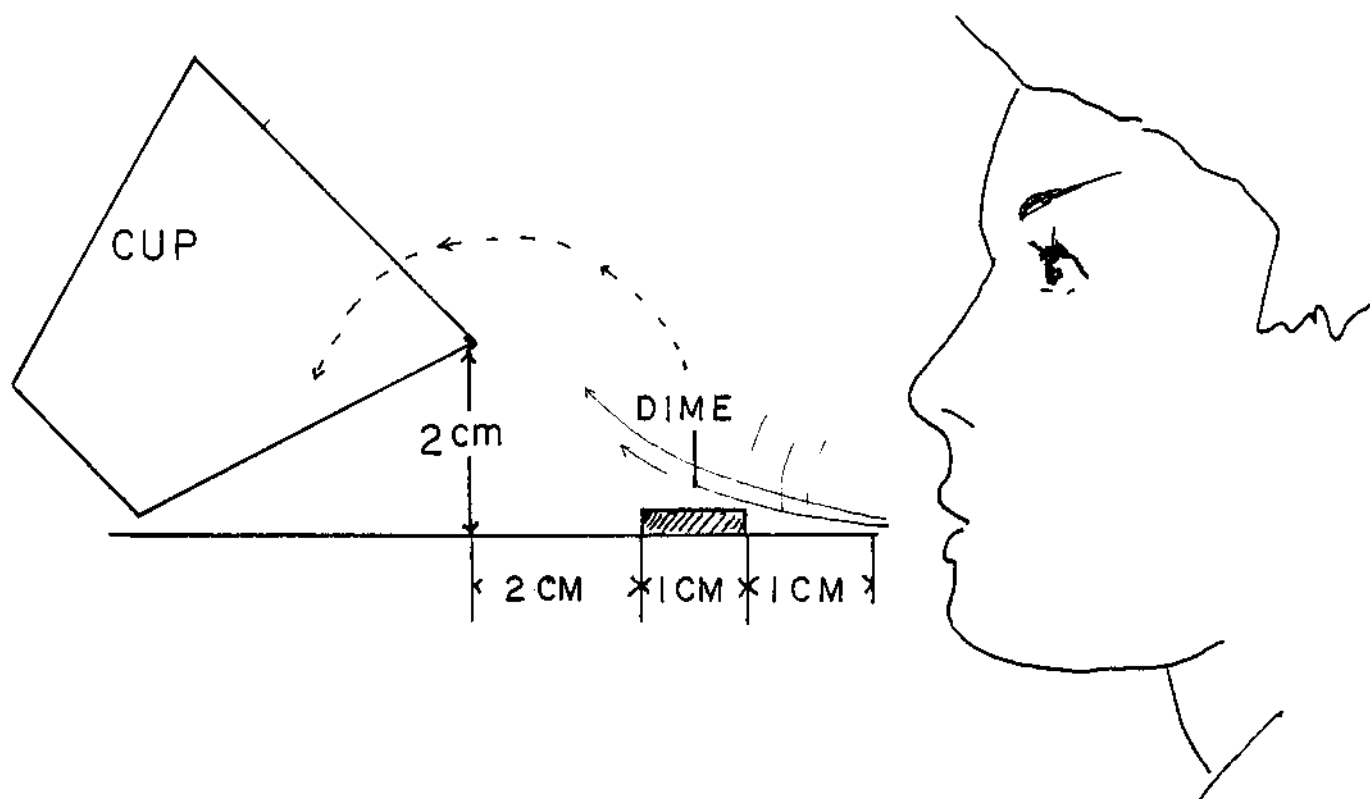
where ρ is the density of air (1 Kg/m^3) and V is the velocity of the air blown over the dime. Now, the area of the dime A ($2.5 \times 10^{-4} \text{ m}^2$) multiplied by this pressure differential must equal the gravitational attraction on the dime, mg , if it is to rise off the table. Since the mass of the dime is 2.24 gm, the gravitational force is about 0.0224N.

So $mg = A \frac{1}{2} \rho V^2$ and, putting in numbers,

$$0.0224 = 2.5 \times 10^{-4} \times \frac{1}{2} \times 1 \times V^2$$



This gives $V = 13.4 \text{ m/sec}$ which is 48 km/hr or 30 mph. If we perform the same trick with a quarter, we require 16 m/sec or 37 mph, and for a nickel it is 17 m/sec or 38 mph. In fact, it must be much more than this to get the coin into the cup. Our little calculation tells us a surprising fact--we can blow at least 30 to 40 mph--something you can tell any blowhards you know!



Experiment 3.14: Viscosity

Materials required: Styrofoam cups, hot and cold water, drinking straw.

Procedure: Viscosity is an interesting subject for simple experiments. When a liquid flows over a fixed surface S (fig. 1) a layer of the solution in a plane parallel to, and a distance ℓ from S flows with a velocity greater than layers close to S . As a result of this relative motion

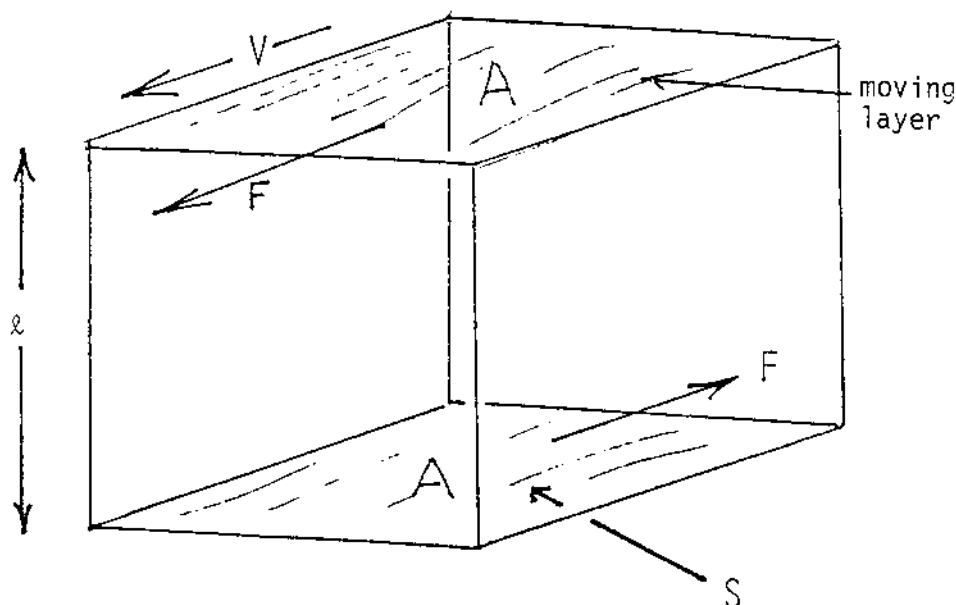


Figure 1

of layers, internal friction or viscosity arises, and the coefficient of viscosity η is given by $\eta = \frac{F \ell}{A v}$ where F is the tangential viscous force between layers of area A distance ℓ apart moving with relative velocity v . The viscosity of water is 1.8 centipoises (the poise is the c.g.s. unit-gm/(sec Cm)) at 0°C , 1 centipoise at 20°C , .5 centipoises at 50°C and .28 centipoises at 100°C . Take a styrofoam cup and poke a small hole in the bottom with a fine needle or pencil tip. Fill the cup with hot water and see how long it takes to empty. Now do the same with cold (preferably iced) water. Since the change in viscosity is about a factor of six, one might anticipate the water would leak out much more rapidly for the hot water--yet it does not. Why is this? The flow through the small hole is turbulent and not streamline. A dimensionless number, called the "Reynolds' number" determines the velocity at which, for a given geometrical configuration, streamline flow ceases. Turbulent flow has a very weak dependence on viscosity, depending mostly on the liquid density. This is the reason the Greek Klepsydra or water clocks required no thermal compensation--turbulent flow ensured that the water dripped at the same rate, more or less independent of temperature.

Another interesting example of turbulent flow can be seen with a toy balloon. Drop an air filled balloon. It takes several seconds to reach the floor. Stokes' formula for the force F on a spherical object of radius a traveling with velocity v through a medium of viscosity η is given by

$$F = 6 \pi \eta a v$$

This formula applies only if the flow is non-turbulent, i.e. streamlined around the sphere.